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Rules of elasticity.

If f and g are differentiable functions of x and A is a constant, then the following rules hold.

$El_x A = 0$.

$El_x (f \cdot g) = El_x f + El_x g$

$El_x (f/g) = El_x f - El_x g$

$El_x (f+g) = \frac{f El_x f + g \cdot El_x g}{f+g}$

$El (f-g) = \frac{f \cdot El_x f - g\, El_x g}{f-g}$.

$El_x f(g(x)) = El_u f(u)\, El_x u$
(where $u = g(x)$)

Continuity.

f is cont @ $x = a$ if $\lim_{x \to a} f(x) = f(a)$

Conditions for continuity

1. The function f must be defined @ $x = a$
2. The limit of $f(x)$ as $x \to a$ must exist.
3. The limit must be exactly equal to $f(a)$

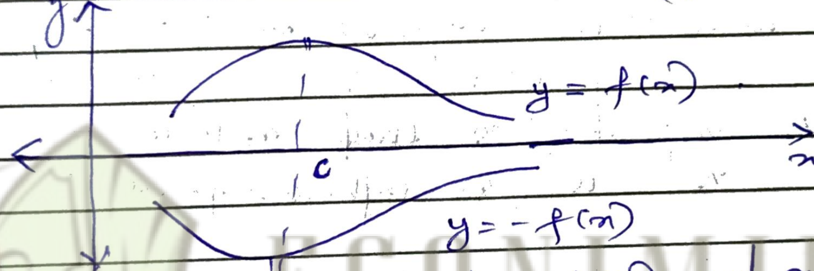
Single Variable Optimization.

x_0 is a stationary point for f if $f'(x_0) = 0$.

If $f(x)$ has domain D.

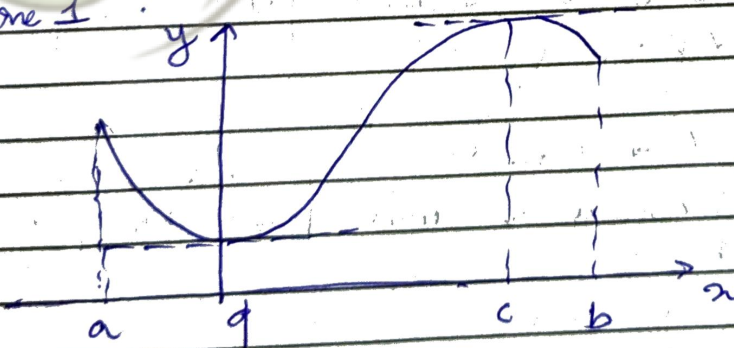
$c \in D$ is a (strict) max pt for $f \Leftrightarrow f(x) \leq f(c)\ \forall x \in D$
$\Downarrow \rightarrow$ $(f(x) < f(c)\ \forall x \in D)$

$d \in D$ is a (strict) min pt for $f \Leftrightarrow f(x) \geq f(d)\ \forall x \in D$
$\Downarrow \rightarrow$ $(f(x) > f(d)\ \forall x \in D)$



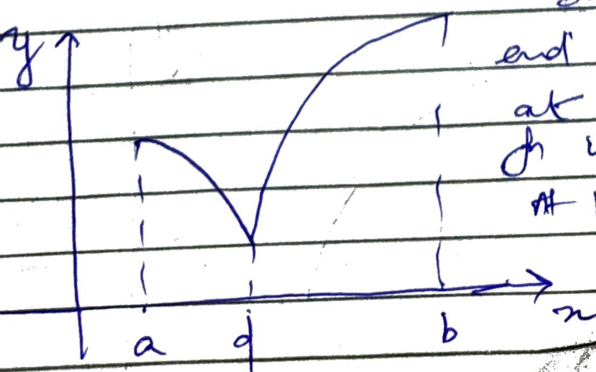
Eg: c is the max pt for $f(x)$ and min pt for $-f(x)$.

Case 1 :

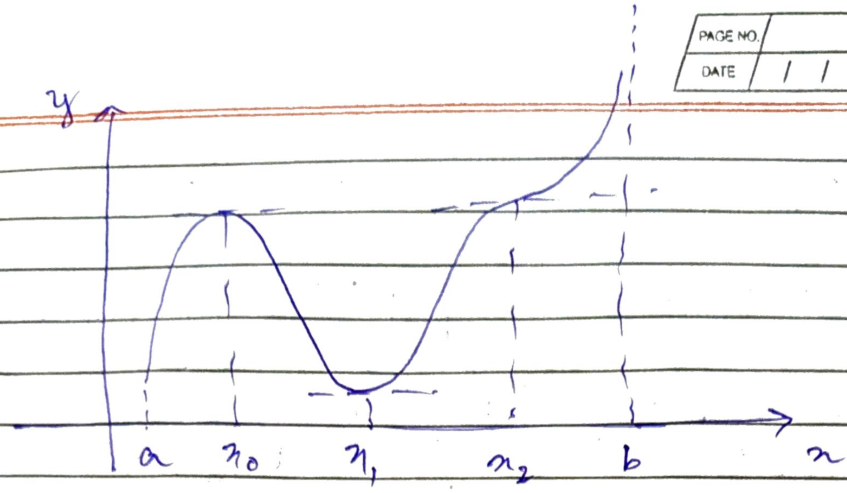


2 stat. pts – d & c.
d – min
c – max.

Case 2



There is a max at end pt b and min at d. At d, the fn is not differentiable. At b the left hand derivative is not 0.



3 Stat. pts — x_0, x_1, x_2.

At end pt a there is a min.

f doesn't have a max as $f \to \infty$ as $x \to b$.

At x_0, f has local maximum and

At x_1 f has local minimum.

x_2 is an inflection point.

First derivative test for extreame points.

If $f'(x) \geq 0 \;\forall\; x \leq c$ and $f'(x) \leq 0$ for $x \geq c$ then $x = c$ is a max point for f. (See fig A).

If $f'(x) \leq 0 \;\forall\; x \leq c$ and $f'(x) \geq 0$ for $x \geq c$ then $x = c$ is a min point for f. (See fig B).

Fig A

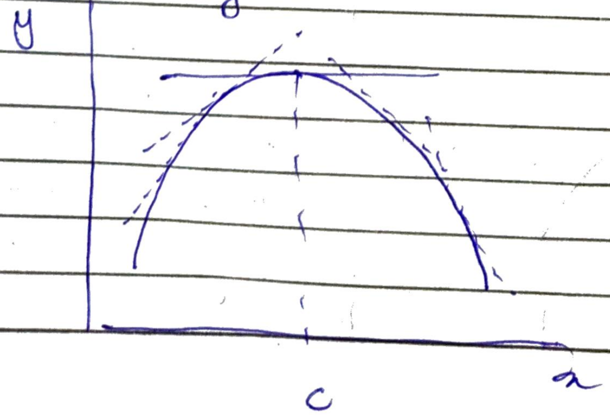
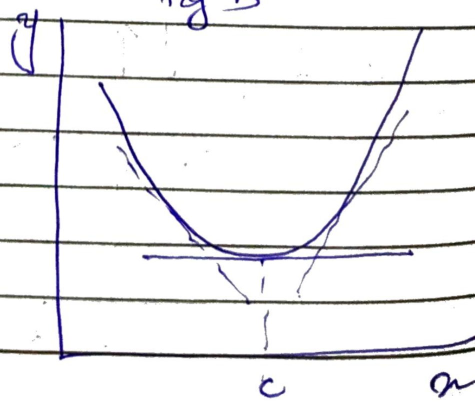


Fig B



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Local Maxima and Minima (1st derivative test):

If $f(x)$ is defined on domain A, then

(i) Function f has a local maximum at c if there is an interval (α, β) about c such that $f(x) \leq f(c)$ $\forall$ x in A that also lie in (α, β)

(ii) Function f has a local minimum at d if there is an interval (α, β) about d such that $f(x) \geq f(d)$ $\forall$ x in A that also lie in (α, β)

Second derivative test:

Let f be a twice differentiable function in an interval I. Suppose c is an interior point of I. Then:

(a) $f'(c) = 0$ and $f''(c) < 0 \Rightarrow c$ is a strict local maxima

(b) $f'(c) = 0$ and $f''(c) > 0 \Rightarrow c$ is a strict local min.

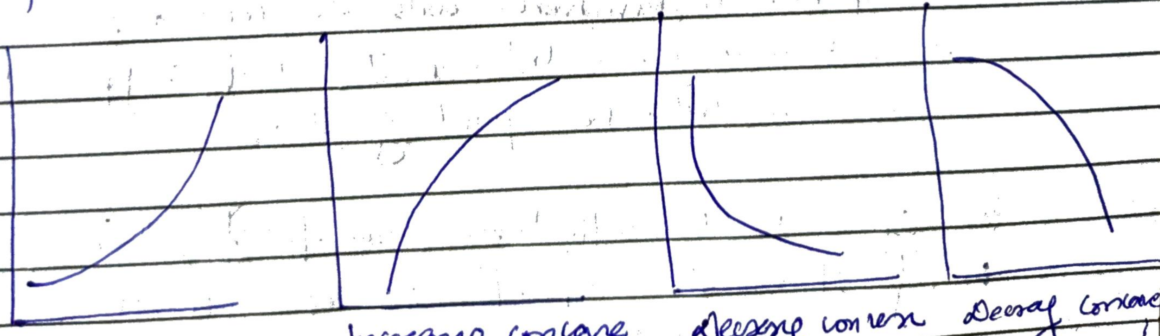
(c) $f'(c) = 0$ and $f''(c) = 0 \Rightarrow c$ is an inflection pt.

Convex and Concave functions:

Let f is continuous in interval I and twice diff in interior of I (I°)

min in I° — f is convex on $I \iff f''(x) \geq 0$ $\forall$ x in I°

max in I° — f is concave on $I \iff f''(x) \leq 0$ $\forall$ x in I°.



Increasing convex — Increasing concave — Decreasing convex — Decreasing concave

Inflection points:

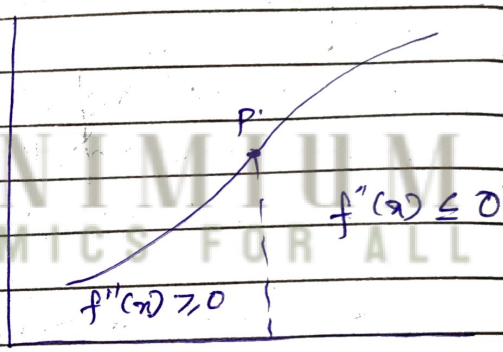
Point c is an inflection point for a twice differentiable function f if there is an interval (a,b) containing c such that either of the following two conditions hold:

(a) $f''(x) \geq 0$ if $a < x < c$ and $f''(x) \leq 0$ if $c < x < b$

or (b) $f''(x) \leq 0$ if $a < x < c$ and $f''(x) \geq 0$ if $c < x < b$.

Briefly $x = c$ is an inflection point if $f''(x)$ changes sign at c.

P is an inflection point



Test for inflection points:

Let f be a function with a continuous second derivative in an interval I, and suppose that c is an interior point of I.

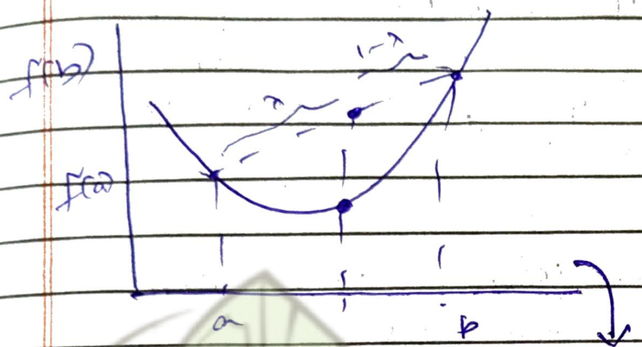
(a) If c is an inflection point for f, then $f''(c) = 0$.

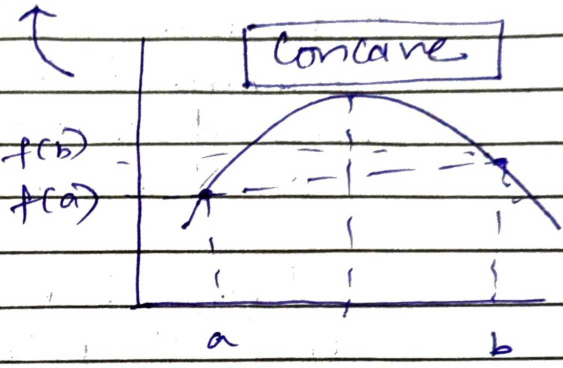
(b) If $f''(c) = 0$ and f'' changes sign at c then c is an inflection point for f.

Important result:

Function f is concave in the interval I if $\forall\ a, b \in I$ and all $\lambda \in (0,1)$.

$$f((1-\lambda)a + \lambda b) \geq (1-\lambda) f(a) + \lambda f(b).$$





Function f is convex if $-f$ is concave

Thus:

f is convex if

$$f((1-\lambda)a + \lambda b) \leq (1-\lambda) f(a) + \lambda f(b)$$

Jensen's inequality (discrete version).

A function f is concave in the interval I iff the following inequality is satisfied $\forall\ x_i \in I$ and $\forall\ \lambda_i \geq 0$ with $\sum_{i=1}^{n} \lambda_i = 1$.

$$f(\lambda_1 x_1 + \cdots \lambda_n x_n) \geq \lambda_1 f(x_1) + \cdots \lambda_n f(x_n).$$