Logic and Set Theory | Mathematical Methods of Economics

Necessary and Sufficient conditions

P is a sufficient condition for Q means P => Q (P implies Q)
Q is a sufficient condition for P means P=> Q (P implies Q)
P is a necessary and sufficient condition for Q means P<=>Q (P iff Q)

Mathematical Proof

Usually it is most natural to prove a result , say P=>Q by starting with the premises P and successively working forward to the conclusion Q, called a direct proof.
Sometimes however it is more convenient to prove the implication P=> Q by an indirect proof. In this case, we begin by supposing Q is not true and on that basis demonstrate that neither P can be true.
Then there is a third method of proof that is also useful called proof by contradiction. The method is based upon fundamental logical principle: that it is impossible for a chain of valid inferences to proceed from a true proposition to a false one. Therefore, if we have a proposition R and we can derive a contradiction on the basis of supposing that R is false, then it follows that R must be true.

Set Theory

Basics

$A \cup B = \{x: x \in A \text{ or } x \in B$
$A \cap B = \{x: x \in A \text{ and } x \in B$
$A|B = \{\{x: x \in A \text{ or } x \notin B$
$A \subseteq B = \{\{x: x \in A \Rightarrow x \in B$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$A \triangle B = (A|B) \cup (B|A)$
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
If $A \cap B = \varphi$, then $n(A \cup B) = n(A) + n(B)$
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$
$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Commutative law

$A \cap B = B \cap A$
$A \cup B = B \cup A$

Associative law

$A \cap (B \cap C) = (A \cap B) \cap C$
$A \cup (B \cup C) = (A \cup B) \cup C$

Distributive law

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Idempotent law

$A \cap A = A$
$A \cup = A$

Law of U and φ

$A \cap \varphi = A$
$A \cup \varphi = A$
$U \cap A = A$
$U \cup A = U$

Necessary and Sufficient conditions

Compliments law

$A \cup A' = U$

$A \cap A' = \varphi$

$A' = U - A$

De- Morgan's Law

$(A \cup B)' = A' \cap B'$

$(A \cap B)' = A' \cup B'$

Law of double compliments

$(A')' = A$

Compliments between U and φ

$U' = \varphi$

$\varphi = U$

$n(A|B) = n(A \cup B) - n(B)$

$\quad\quad = n(A) - n(A \cap B)$

$n(A') = n(U) - n(A)$

Power Set

It is a collection of all subsets of say, set A

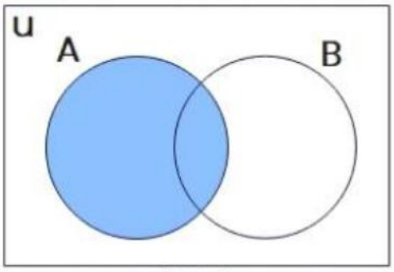
If A={p,q), then P(A)= {φ, {p}, {q}, {p,q}}

$n(P(A)) = 2^m$ where $n(A) = m$

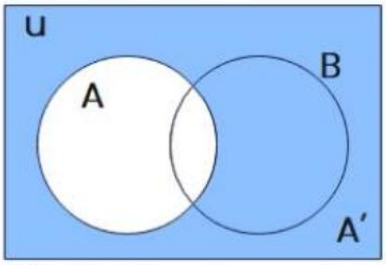
Proper subset [⊂]

Set A is considered to be a proper subset if Set B contains at least one element that is not in A. If we have to pick n number of elements from a set containing N elements, it can be done in ***NCn*** ways which also becomes the number of possible subsets containing n number of elements from a set with cardinality N. If a set has n elements, the number of subsets of the given set is 2^n and number of proper subsets would be 2^{n-1}

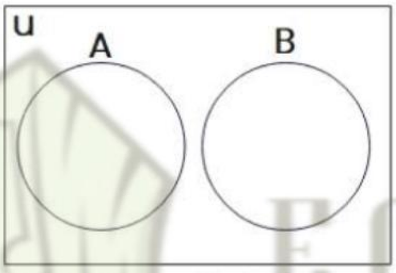
Venn Diagrams



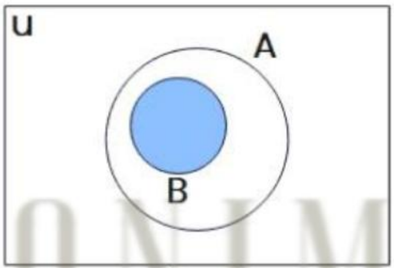
Set A



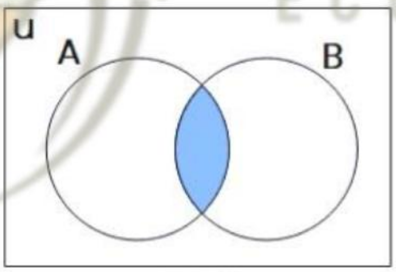
A′is the Complememt of A



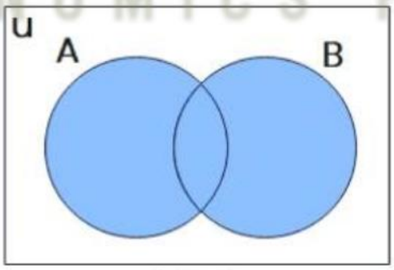
A & B are disoint sets



$B \subset A$
B is the proper subset of A



$A \cap B$
A & B are overlapping sets



$A \cup B$
A & B are overlapping sets