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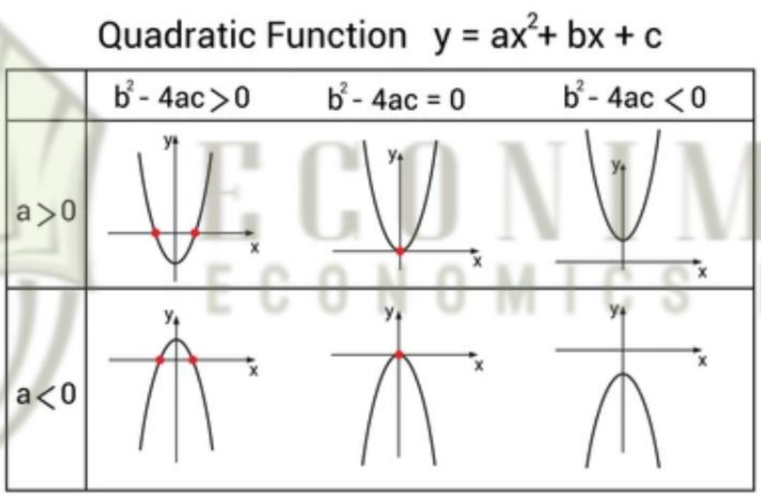
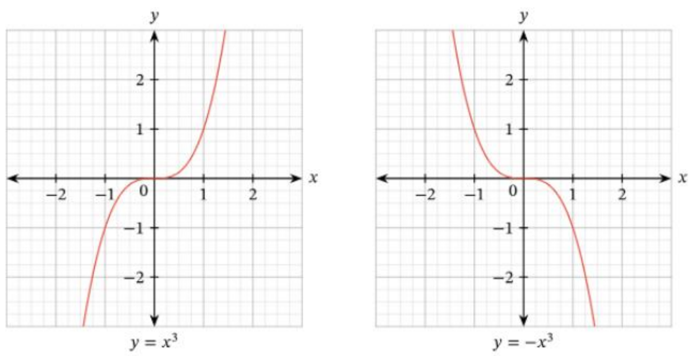
Quadratic functions

- $y=ax^2+bx+c$
- $x=[-b\pm\sqrt{(b^2-4ac)}]/2a$
- If $b^2-4ac=0$, then the function has one zero i.e. $x=-b/2a$
- If $b^2-4ac>0$, then 2 zeroes.
- If $b^2-4ac<0$, then no zeroes.
- If $a>0$, $f(x)$ attains min at $-b/2a$ and min value is $c-b^2/4a$
- If $a<0$, $f(x)$ attains max at $-b/2a$ and max value is $c-b^2/4a$



Cubic functions

- $f(x)= ax^3+bx^2+cx+d$



General polynomials

- $P(x)= a_n x^n + a_{n-1}x^{n-1}+...............+a_1x+ a_0$ is called a general polynomial of degree n. Often one is interested in finding the number and location of the zeroes of $P(x)$ i.e. where $P(x)=0$
- $a_n x^n + a_{n-1}x^{n-1}+...............+a_1x+ a_0 =0$ is called the general nth order equation.

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- Note:- According to fundamental theorem of algebra, every polynomial of the general form can be written as a product of first or second degree.

Remainder Theorem

- Let $P(x)$ and $Q(x)$ be two polynomials for which the degree of P(x) is greater than or equal to the degree of $Q(x)$. Then, there always exist unique polynomials $q(x)$ and $r(x)$ such that

$$P(x)=q(x)\,Q(x)+r(x) \qquad (\#)$$

where the degree of $r(x)$ is less than degree of $Q(x)$. This fact is called the remainder theorem.

- When x is such that $Q(x) \neq 0$, then $(\#)$ can be written as

$$P(x)/Q(x) = q(x) + r(x)/Q(x)$$

- If $r(x)=0$, we say that $Q(x)$ is a factor of $P(x)$ or that $P(x)$ is divisible by $Q(x)$

  i.e. $P(x)/Q(x)= q(x)$ which is the quotient when $r(x) \neq 0$, is the remainder.

- An important special case is when $Q(x)=x-a$. Then $Q(x)$ is of degree 1, so $r(x)$ must have degree 0 and therefore is a constant. In that case,

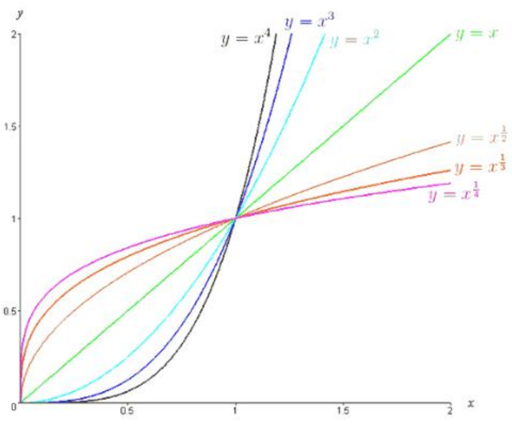
$$P(x)= q(x)(x-a) + r \quad \forall x$$

- For $x=a$ in particular, we get $P(a)=r$. Hence, $x-a$ divides $P(x)$ iff $P(a)=0$

  i.e. $P(x)$ has a factor $x-a \iff P(a) =0$

Power Functions

- $f(x)=x^r$



Property	Symbols
Product of Powers	$b^m \cdot b^n = b^{m+n}$
Power of a Power	$(b^m)^n = b^{m \cdot n}$
Power of a Product	$(ab)^m = a^m b^m$
Quotient of Powers	$\frac{b^m}{b^n} = b^{m-n}$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
Zero Exponents	$b^0 = 1$
Negative Exponents	$b^{-m} = \frac{1}{b^m}$
Rational Exponents	$b^{\frac{p}{r}} = (\sqrt[r]{b})^p$