



Relations, Functions and Graphs | Mathematical Methods of Economics

Relations

Cartesian Product

$$A \times B = \{ (a, b) : a \in A, b \in B \}$$

$$A \times B \neq B \times A$$

$$n(A \times B) = mn \text{ if } n(A)=m \text{ and } n(B)=n$$

Relation

A relation from A to B is a subset of $A \times B$ i.e. $R \subseteq A \times B$

Number of relations from A to B is 2^{mn}

Universal relation

It is the largest relation i.e. $A \times B$

Void relation

$$R = \varnothing \subseteq A \times A$$

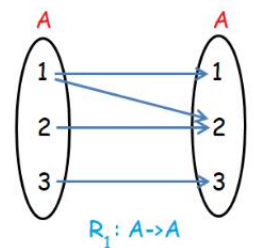
Identity relation

Every element of A is related to itself

Reflexive relation

If $(a, a) \in R \quad \forall a \in A$, then every element of A is related to itself. (see fig)

$$\text{Number of reflexive relations} = 2^{(n \wedge 2) - n}$$



Symmetric relation

Relations where if $aRb \Rightarrow bRa$

eg: $P = \{(2,3), (3,2), (1,1)\}$; P is symmetric

Note: Empty relation is a symmetric set always.

Transitive relation

Relations where if aRb and bRc then aRc

Equivalence relation

A relation that is reflexive, symmetric and transitive



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Functions

A relation from A to B is a function if every element of A is related to a unique element in B.

Types of Functions

One-One functions (Injective)

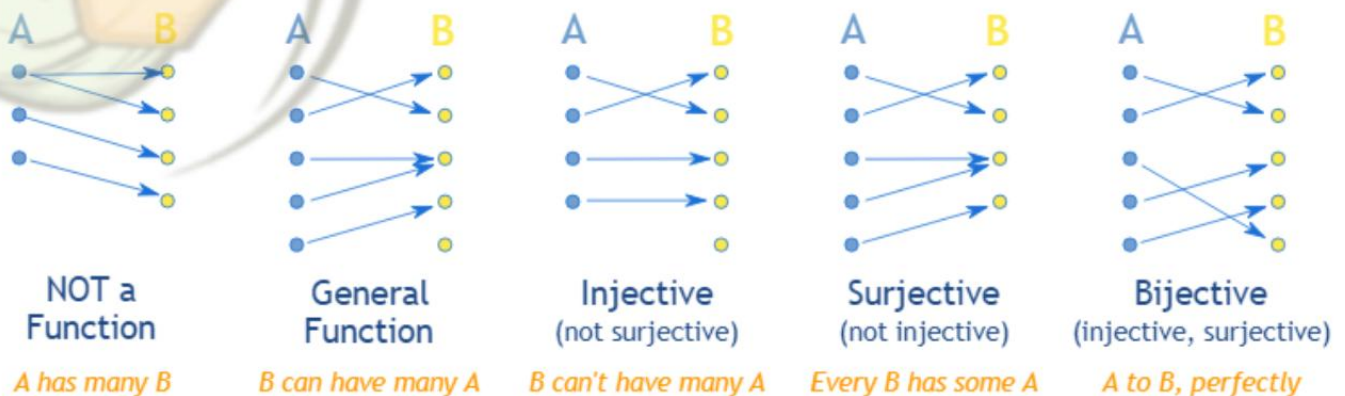
Cases where different elements of A have different images in B

Onto function (Surjective)

Cases where if every element $b \in B$ has at least one pre-image $a \in A$

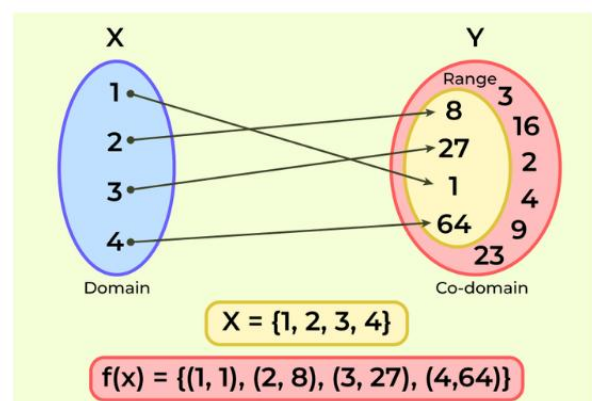
Bijjective function

Functions which are both one-one and onto



Functions of one real variable

A function of a real variable x with domain D is a rule that assigns a unique real number to each number x in D .



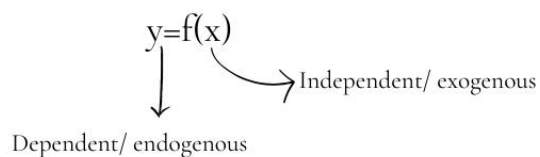


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Domain and Range of some commonly used functions:

Function	Domain	Range
Polynomial function	R	R
Identity function x	R	R
Constant function K	R	$\{K\}$
Reciprocal function $\frac{1}{x}$	R_0	R_0
$x^2, x $	R	$R^+ \cup \{0\}$
$x^3, x x $	R	R
Signum function	R	$\{-1, 0, 1\}$
$x + x $	R	$R^+ \cup \{0\}$
$x - x $	R	$R^- \cup \{0\}$
$[x]$	R	I
$x - [x]$	R	$[0, 1)$
$\sqrt{x}$	$[0, \infty)$	$[0, \infty]$
a^x	R	R^+
$\log x$	R^+	R
$\sin x$	R	$[-1, 1]$
$\cos x$	R	$[-1, 1]$

If f is a function, we often denote y as the value of f at x as



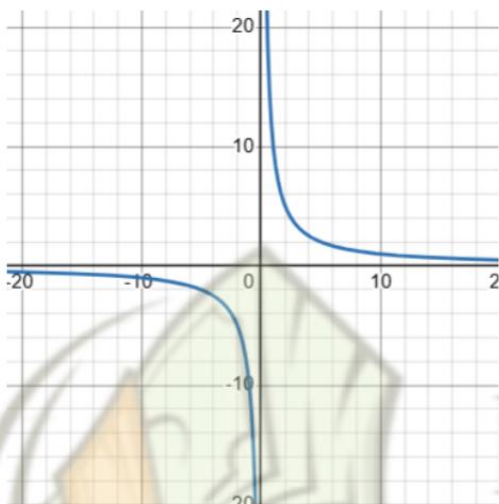


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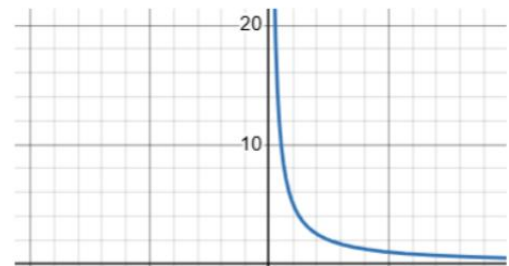
Graphs- Basics

Here are some basic general forms of functions and its graphs:

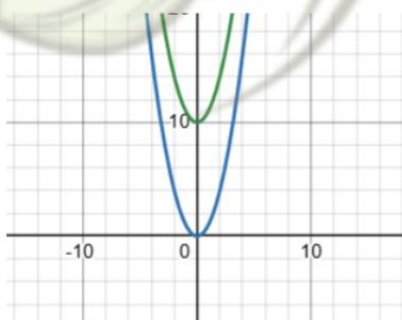
$$xy=c$$



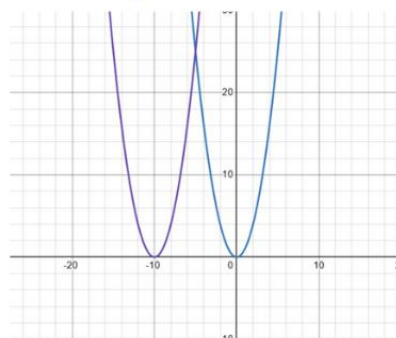
$$xy=c, x>0, y>0$$



$$y=f(x)+c$$

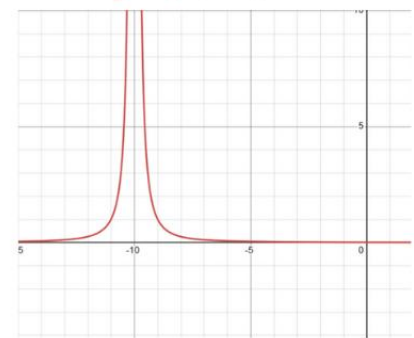


$$y=f(x)$$



$$y=f(x+c)$$

$$y=1/f(x+c)$$



$f(x)=x^2$ and $c=10$ in these examples

Linear function

It is of the general form $y=a+bx$ where a is the intercept and b being the slope.

The figure alongside showcases an increasing linear function with $a=4$ and $b=4/3$

