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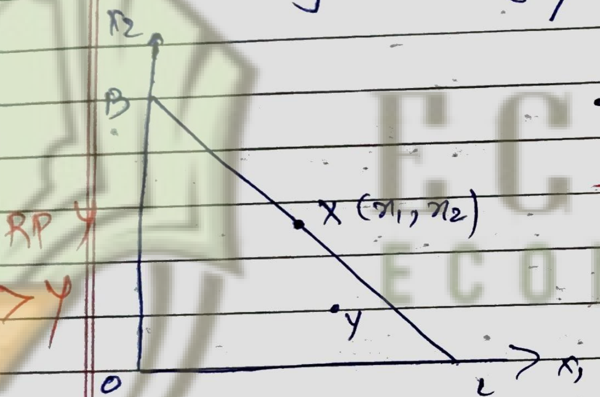
Ch 7 :

Revealed Preference

Up until now, we were thinking about what preferences could tell us about peoples behaviour. But in real life, preferences are not-directly observable : we're to discover people's preferences from observing their behavior.

The assumption of people's preferences remaining unchanged forever isn't true.



Take X as the chosen bundle. What is true about the situation is that, X was chosen when Y was available. In fact X was chosen over all other bundles on BL and beneath BL $\in$ OBL.

Bundle Y is affordable but not demanded.

Before moving forward with the concept of revealed preferences and the principle, let's talk abt convexity of pref.

Consider a bundle (x_1, x_2) - X

Let $S_x = \{y : y \succsim x\}$

S_x is convex $\Leftrightarrow$

$y, z \in S_x$ then

$\alpha y + (1-\alpha) z$

'Strict convexity is where S_x is strictly convex

$\Rightarrow$ $y, z \in S_x$ then

$\alpha y + (1-\alpha) z \in S_x$, $\alpha \in (0,1)$

* Convexity of preferences rules out the straight line IC makes it a smooth convex curve so that there exist only a unique demanded bundle.

and ICs with flat areas.

Direct Revealed Preference

A bundle X is said to be directly revealed pref to another bundle Y if X was chosen / demanded (equilibrium) bundle when Y was available.

Y was available $\Rightarrow$ $P_1 y_1 + P_2 y_2 \leq m$

X was demanded $\Rightarrow$ $P_1 x_1 + P_2 x_2 = m$

from ① & ②

$$P_1 x_1 + P_2 x_2 \geq P_1 y_1 + P_2 y_2$$, This is

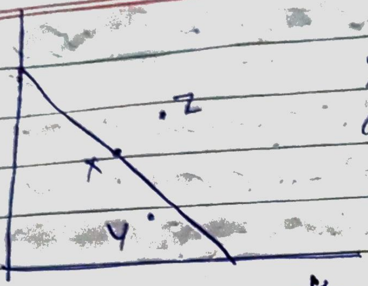
the first property ie X is atleast as costly as Y,

Also the strict convexity assumption gives property 2 ie $(x_1, x_2) \succ (y_1, y_2)$.

'Revealed pref $\Longrightarrow$ Preference'

ie x DRP y $\Rightarrow x \succ y$

Not the other way around; because there are cases where a set is preffered but is not revealed pref



X is the chosen bundle.
X was chosen when Y was available.
$\therefore$ X DRP Y
$\Rightarrow X \succ Y$.
the other way? Take X and Z.
$Z \succ Y \not\Rightarrow$ Z DRP X as Z is unaffordable.

For DRP, preference with "affordability" is taken into consideration. For 'preference' there is no role for affordability.

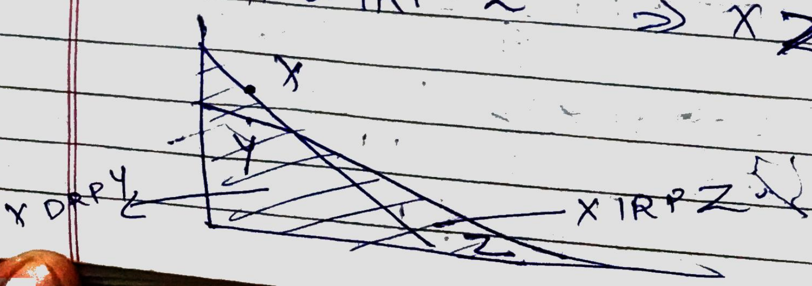
The principle of Revealed preference

Let (x_1, x_2) be the chosen bundle when prices are (p_1, p_2) and let (y_1, y_2) be some other bundle such that $p_1x_1 + p_2x_2 \geq p_1y_1 + p_2y_2$. Then if the consumer is choosing the most preferred bundle she can afford, we must have $(x_1, x_2) \succ (y_1, y_2)$

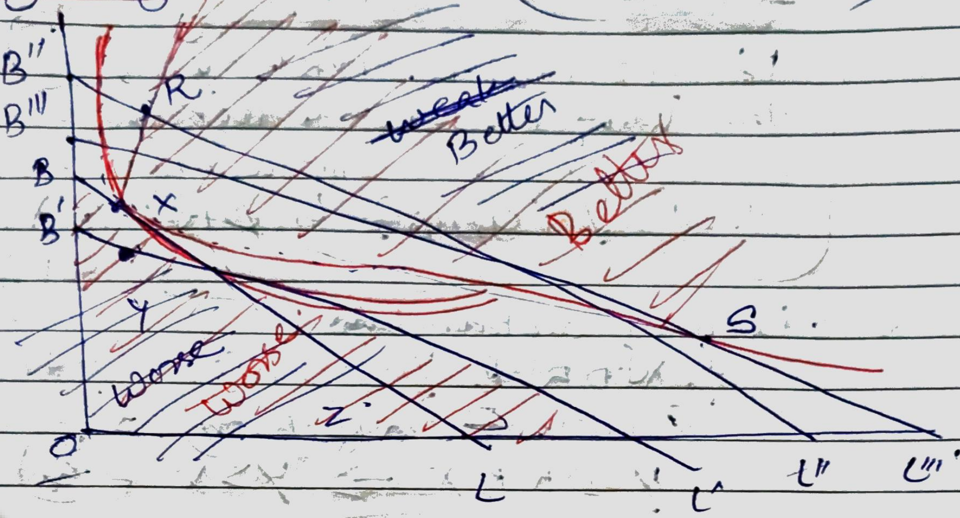
Indirect Revealed preference.

If X DRP Y $\Rightarrow X \succ Y$
Y DRP Z $\Rightarrow Y \succ Z$
then X is IRP Z $\Rightarrow X \succ Z$

(because of transitivity of preference not 'drp'.)



Recovering preferences. (Refer Fig 7.3)



In OBL
~~X RP Y~~

X RP Y ⇒ X > Y ⇒ X > Z

Y RP Z ⇒ Y > Z Thus every bundle under OBL and OB'L' is worse than X.

Take OB''L''

R is the chosen bundle.

ie R RP X ⇒ R > X

Take OB'''L'''

S is the chosen bundle

ie S RP X ⇒ S > X

Take a line RX.

any bundle ∈ (RX) 'll be strictly preff over X due to strict convexity assumption.

With multiple data points we are able to put restrictions on the preference that are pref over X. Though an absolute mapping isn't possible.

WARP — Weak Axiom of Revealed preference.

If $X(x_1, x_2)$ is DRP $Y(y_1, y_2)$ and the two bundles are not the same (i.e $X \neq Y$) then it cannot happen that $Y(y_1, y_2)$ is DRP to $X(x_1, x_2)$.

Proof by contradiction

X DRP $Y \Rightarrow X > Y$ — (1)

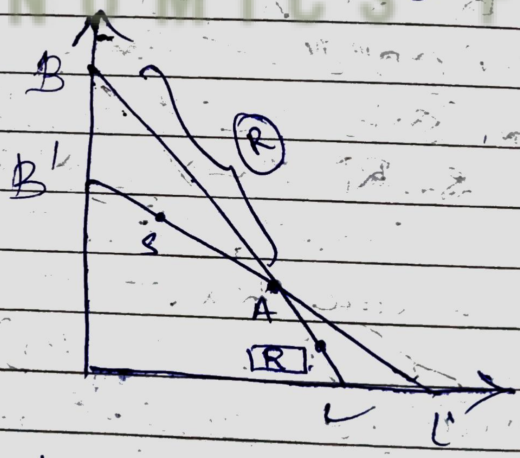
Suppose $\exists$ a (p_1, p_2, m) s.t.

Y DRP $X \Rightarrow Y > X$ — (2)

from (1) & (2) $\Rightarrow$ contradiction.

However if X DRP Y, then it must be the case that X bundle is unaffordable if Y was bought.

(A)
- R is chosen when BL
- S is chosen when B'L'



When BL, $\boxed{R}$ is chosen when S was available — (1)

When B'L' S is chosen when R was available, — (2)

(1) & (2) $\Rightarrow$ Violation of WARP.

So what can be done to make [A] true without violating WARP?

It happens iff $\boxed{R}$ becomes (R) i.e. S is chosen when R was no longer available. This happens when (R) outside $OB'C'$. The violation continues for all points within $OB'C'$ like $\boxed{R}$.

As long as R is b/w B and A, WARP'll not be violated and $\boxed{A}$'ll be true.

At (P_1, P_2) when R is chosen
Exp is $P_1r_1 + P_2r_2$
when S is chosen
Exp $P_1s_1 + P_2s_2$

Note $P_1s_1 + P_2s_2 < P_1r_1 + P_2r_2$ for [A] to be true

Checking WARP

P_1 (1,2) B_1 (1,2)
P_2 (2,1) B_2 (2,1)
P_3 (1,1) B_3 (2,2)

	B_1	B_2	B_3
P_1	5	4	6
P_2	4	5	6
P_3	3	3	4

Clearly
B_1 DRP B_2
B_2 DRP B_1 } These are violations.
B_3 DRP B_1
B_3 DRP B_2.

It is because either the consumer is not choosing the best bundle or there is some other aspect of the choice problem that has changed that we have not observed.

Remember WARP is a necessary condition for a Rational consumer
i.e. ~~RB~~. Rationality $\Rightarrow$ WARP holds.

SARP - Strong Axiom of Revealed preference

Remember (defn) X RP Y iff X DRP Y or X IRP Y.

SARP:- If (x_1, x_2) is revealed pref to (y_1, y_2) (either directly or indirectly) and (y_1, y_2) is diff from (x_1, x_2) then (y_1, y_2) can't be directly or indirectly revealed pref to (x_1, x_2).

(Proof to be of 2 parts :- Proof of contradiction for DRP part & IRP in similar way as way the case for WARP)

Remember SARP is a sufficient condition
i.e. SARP $\Rightarrow$ RC !

Checking SARP :

	B_1	B_2	B_3
P_1	20	10	22
P_2	21	20	19
P_3	12	15	10

B_1 DRP B_2 $\Rightarrow$ B_1 RP B_2
B_2 DRP B_3 $\Rightarrow$ B_2 RP B_3
B_1 IRP B_3 — B_1 RP B_3.

No violations.

Index Numbers.

Suppose we examine the consumption bundles of a consumer at 2 diff. times and we want to compare how consumption changed. We use indexes

b - base period ; t - period t ; p - price x - bundle.

Quantity index (I_q)

If we let w_1 and w_2 be some weights that go into making an average, then we can look at the follwg. qty index

$$I_q = \frac{w_1 x_1^t + w_2 x_2^t}{w_1 x_1^b + w_2 x_2^b},$$

(x_1^t, x_2^t) bought when (p_1^t, p_2^t)

(x_1^b, x_2^b) bought when (p_1^b, p_2^b)

A natural choice to keep prices as weights

If we use base period prices for the weights, we've something called a Laspeyres index and if used are the 't' period prices, we've something called paasche index

Paasche qty index

$$P_q = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^t x_1^b + p_2^t x_2^b}$$

Laspeyres quantity index

$$L_q = \frac{p_1^b x_1^t + p_2^b x_2^t}{p_1^b x_1^b + p_2^b x_2^b}$$

The magnitudes of indexes

Consider (x_1^t, x_2^t) the chosen bundle.

P_q: $\boxed{\text{Take } P_q > 1}$

$$\Rightarrow p_1^t x_1^t + p_2^t x_2^t > p_1^b x_1^b + p_2^b x_2^b$$

Thus it shows that (x_1^t, x_2^t) was chosen when (x_1^b, x_2^b) was available.

Thus (x_1^t, x_2^t) RP (x_1^b, x_2^b).

Take $P_q < 1$.

$$\Rightarrow p_1^t x_1^t + p_2^t x_2^t < p_1^b x_1^b + p_2^b x_2^b$$

Here we can see that (x_1^b, x_2^b) is unaffordable and (x_1^t, x_2^t) was chosen. No conclusions of revealed preference can be drawn.

Take $L_q < 1$

Let (x_1^b, x_2^b) be the chosen bundle

$$\Rightarrow p_1^b x_1^t + p_2^b x_2^t < p_1^b x_1^b + p_2^b x_2^b .$$

$$\Rightarrow (x_1^b, x_2^b) \ RP \ (x_1^t, x_2^t) .$$

$L_q > 1 \Rightarrow$ no conclusion can be drawn.

Price Index (I_p)

Price index 'll be a weighted avg of prices

$$I_p = \frac{p_1^t w_1 + p_2^t w_2}{p_1^b w_1 + p_2^b w_2}$$

Qty can acts as the weights.

If we chose the base period ~~we~~ qties we get Laspeyres price index and if we use that of period 't' we'll get Pasche price index.

Pasche price Index

$$P_p = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^t + p_2^b x_2^t}.$$

Laspeyres price Index

$$L_p = \frac{p_1^t x_1^b + p_2^t x_2^b}{p_1^b x_1^b + p_2^b x_2^b}.$$

No comparison with revealed preferences can be made as these are now diff prices in the numerator ~~and~~.

Index of change in total expenditure (M).

$$\boxed{M = \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b}.}$$

This is the ratio of total exp in period t to the total expenditure in period b.

Take $\boxed{P_p > M}$. b year bundle is chosen

ie $$\frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^t + p_2^b x_2^t} > \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b}$$

$$\Rightarrow p_1^b x_1^b + p_2^b x_2^b \geq p_1^b x_1^t + p_2^b x_2^t.$$

This says that the base year bundle is revealed preffered to the t period bundle ∴ That means the consumer is better off in base year b than in year t.

If prices rise by more than income rises in the movement from b to t, we would expect that would tend to make the consumer worse off.

$\boxed{\text{Take } L_p < M}$ t year bundle is chosen

i.e $$\frac{p_1^t x_1^b + p_2^t x_2^b}{p_1^b x_1^b + p_2^b x_2^b} < \frac{p_1^t x_1^t + p_2^t x_2^t}{p_1^b x_1^b + p_2^b x_2^b}$$

$$\Rightarrow p_1^t x_1^b + p_2^t x_2^b < p_1^t x_1^t + p_2^t x_2^t.$$

This says that the t year bundle is revealed preff to that of b year bundle. The consumer is better off in 't' year than base year.

If price less than income, the consumer would be better off than before.

* In case of price indices, what matters is not whether the index is greater or less than 1, but whether its greater or less than the expenditure index.

* Indexing Social Security Schemes — Read & Study pg 134.