Unit 4
Utility

The concept of utility was initially thought of as a numeric measure of a person's happiness. The theory of consumer behavior has been reformulated entirely in terms of consumer preferences, and utility is seen only as a way to describe preferences.

~~Cardinal and~~ Ordinal analysis.

There are two analysis of utility.

There is a way of assigning a number to every possible consumption bundle such that more-preff bundle get assigned larger numbers than less-preff bundles.

$$(x_1, x_2) > (y_1, y_2) \text{ iff } u(x_1, x_2) > u(y_1, y_2)$$

Because of the emphasis on ordering the bundles, it is reffered as ordinal utility analysis.

Monotonic transformation (MT)

A monotonic transformation is a way of
* transforming one set of numbers into another set of numbers in a way that preserves the order of the numbers.

MT is represented by $f(u)$.

Remember $u_1 > u_2 \Rightarrow f(u_1) > f(u_2)$.

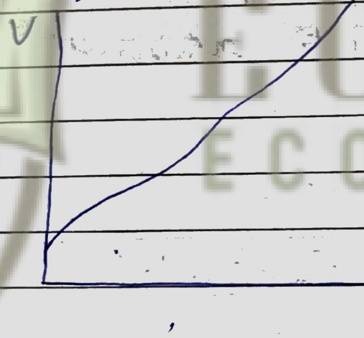
Eg of monotonic transformations are X^n by a +ve no, adding any no, raising u to an odd power.

The rate of change of $f(u)$ as u changes can be measured as:

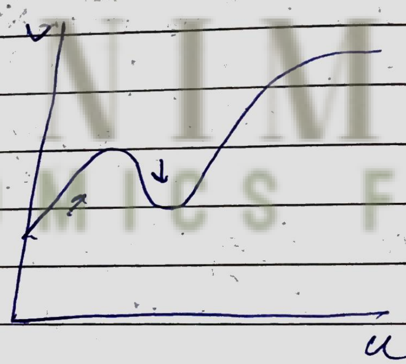
$$\boxed{\frac{\Delta f}{\Delta u} = \frac{f(u_2) - f(u_1)}{u_2 - u_1}}$$

* For a monotonic transformation, $f(u_2) - f(u_1)$ always has the same sign as $u_2 - u_1$.

The MT we are discussing is +ve M.T. -ve MT are such that reverses the order of numbers



A



B

A represents a monotonic fn.
B doesnt represent MT

(Read 57 first para)

* **Note:** A monotonic transf. of a utility fn that represents the same pref. as the original utility fn.

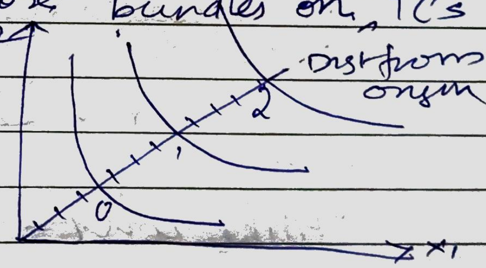
Cardinal Utility:

Theory of utility that assigns a magnitude rather than ordering is known as cardinal utility analysis.

Constructing a Utility function:

We can't always find or define a utility from the preference.
Some example includes perverse cases like intransitive preferences such that $A > B > C > A$. Then utility fn for those pref. 'ld 've to be s. $u(A) > u(B) > u(C) > u(A)$, which is impossible.

If preferences are monotonic, then the line through the origin must intersect every IC exactly once. Thus every bundle is getting a label, and those bundles on higher IC's getting larger labels.



Concept of Marginal Utility

Rate of change in utility (Δu) associated with a small change in the amt of good 1 (Δx_1).

$$MU_1 = \frac{\Delta u}{\Delta x_1} = \frac{u(x_1 + \Delta x_1, x_2) - u(x_1, x_2)}{\Delta x_1}$$

change
w.r.t ch

Thus $\Delta U = MU_1 \cdot \Delta x_1$.

|||ly $\Delta U = MU_2 \cdot \Delta x_2$

It is imp to note that magnitude of MU depends on the magnitude of utility. Also remember MU itself has no behavioural content (read pg 66 last para before 4.5).

Marginal Utility and MRS.

We know MRS is the slope of the IC which is the rate at which a consumer is willing to substitute a small amt of good 2 for good 1.

A utility fn $U(x_1, x_2)$ can be used to measure the MRS.

Consider a change in the cons of each good $(\Delta x_1, \Delta x_2)$ that keeps utility const — ie a change in cons that moves us along the IC. then we must have

$$\boxed{MU_1 \Delta x_1 + MU_2 \Delta x_2 = \Delta U = 0}$$

Solving we get

$$\boxed{MRS = \frac{\Delta x_2}{\Delta x_1} = \frac{-MU_1}{MU_2}}$$

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The -ve sign interprets that if you get more of (say) good 1, you've to get less of good 2 in order to keep the same level of utility.

The utility fn is not uniquely determined. Any monotone transformation of a utility fn leaves you with another equally valid utility fn.

The ratio of MUs is independent of the particular transformation of the utility fn.

i.e. $\boxed{MRS = -\frac{t \cdot MU_1}{t \cdot MU_2} = \frac{MU_1}{MU_2}}$

Thus taking a MT is just relabelling the IC S.

Utility of Commodity - Just read.

Q. Prove $x > y$ and $y > x$ and $x \sim y$ n $y \sim x$.
Q. Prove if $\succsim$ is transitive, then show that $>$ is also transitive.
Q. Is $>$ complete?

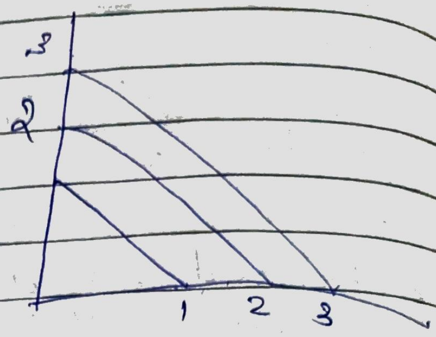
If $u(x)$ represent preferences $\succsim$ then $f(u(x))$ also represents $\succsim$ iff $x \succsim y$ and $u(x) \geq u(y)$

then $f(u(x)) \geq f(u(f(y)))$.

eg: $u(x) = x+y$ (1,1) (2,1)
$f((u(x)) = u^2$ or $2u$.
both gives higher values for (2,1) against (1,1).

Counter example

$f(u(x)) = -2u$



$(1,0), (0,1) \rightarrow$ welfare $= 1$,

$f \rightarrow (1,0)(0,1) - 1$

$(2,0), (0,2)$ welfare $= 2$

$f \rightarrow (2,0), (0,2) = 4$,

f is a diff fn.

eg: f is a fn of u and $f(u)$ is diff.

$f'(u) > 0$

As long as ~~ht~~ you have +ve positive transformation in $f'(u)$ i.e. $f(u) > 0$, then $u(x)$ represents the pref, then $f(u(x))$ also represents the same preference as $u(x)$

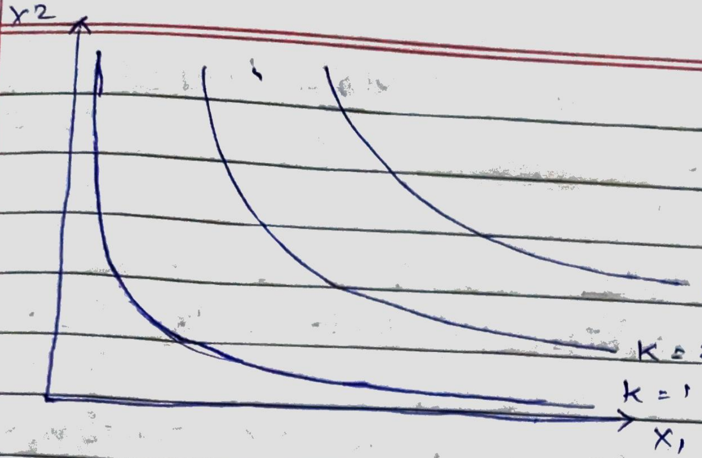
Some examples of Utility functions.

(1) $u(x_1, x_2) = x_1 \cdot x_2$

or in general $x_1^{\alpha} \cdot x_2^{1-\alpha}$

Typical IC are s. $x_1 \cdot x_2 = k$ (level curves) -

$\Rightarrow x_2 = \dfrac{k}{x_1}$



Any the monotonic trns f will give the same kind of image.

(CPN: This is a type of CD fn).

The IC are ↓ sloping convex to origin monotonous curves.

$$MRS_{x_1, x_2} = \frac{MU_{x_1}}{x_2} = \frac{-x_2}{x_1}$$

$$du = \frac{\partial u(\cdot)}{\partial x_1} \cdot dx_1 + \frac{\partial u(\cdot)}{\partial x_2} \cdot dx_2$$

$$0 = dy = \frac{\partial u(\cdot)}{\partial x_1} \cdot dx_1 + \frac{\partial u(\cdot)}{\partial x_2} \cdot dx_2$$

$$\Rightarrow \frac{dx_1}{dx_2} = -\frac{\frac{\partial u(\cdot)}{\partial x_1}}{\frac{\partial u(\cdot)}{\partial x_2}} = -\frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = \frac{MU_{x_1}}{MU_{x_2}}$$

$$\frac{MU_{x_1}}{MU_{x_2}} = MRS_{x_1, x_2}$$

» (2) $u(x_1, x_2) = ax_1 + bx_2$; $|s| = -a/b$

Note: when $a = b = 1$

$u = x_1 + x_2$

This case of u denotes perfect substitutes.

Other usual cases of substitute are $2x_1 + x_2$, $3x_1 + 2x_2$, $x_1 + 5x_2$

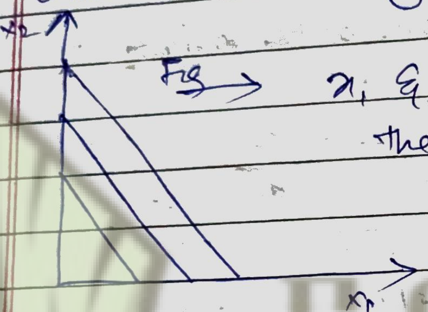


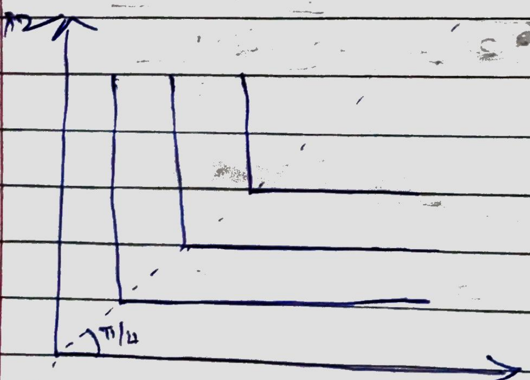
Fig → x_1 & x_2 are perf. substitutes in the ratio 1:1.

$\boxed{MRS_{x_1, x_2} = -1}$

» (3) $u(x_1, x_2) = \min\{ax_1, bx_2\}$

Take the case of ~~red~~ right shoes & left shoes, these. it is a 1:1 ratio complements. Thus

$u = \min(x_1, x_2)$ — perfect complements, which is a particular case.



kink @ $x_1 = x_2$

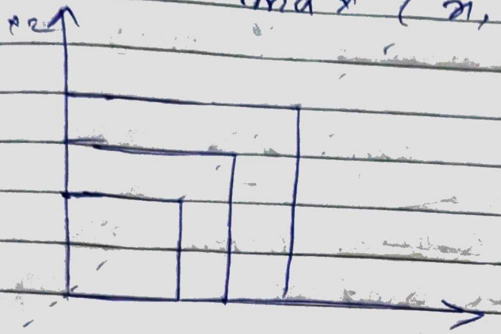
where MRS is not defined

horizontal part, $\boxed{MRS_{x_1, x_2} = 0}$

vertical part, $\boxed{MRS_{x_1, x_2} = \infty}$.

Fig — Perf. compl.

(4) $u = \max(x_1, x_2)$

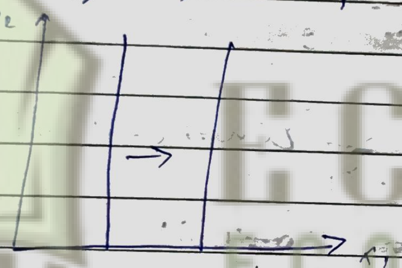


MRS isn't defined on the kinks where $x_1 = x_2$.

horizontal part, $MRS_{x_1,x_1} = 0$

Vertical part, $MRS_{x_1,x_2} = \infty$

(5) $u(x_1, x_2) = x_1 \Rightarrow$ (x_2 is a neutral good).



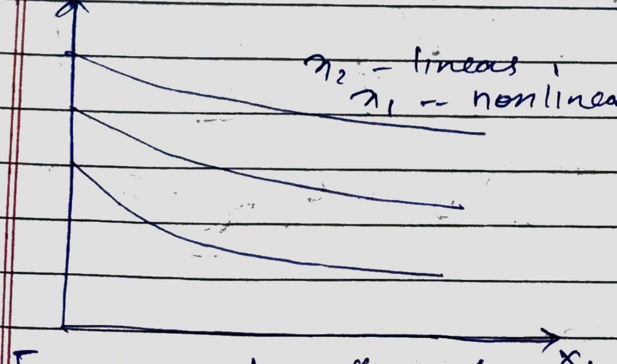
$MRS_{x_1,x_2} = \infty$

(6) $u(x_1, x_2) = v(x_1) + x_2 = \bar{u}$.

This case is called Quasilinear preferences.
In this case both are "goods" $\Rightarrow$ $\downarrow$ sloping ICs.
Here the ICs are vertical translates of one another.

$$x_2 = \bar{u} - v(x_1)$$

$$\boxed{MRS_{x_1,x_2} = \frac{-v'(x_1)}{1}}$$



eg:- $\ln x_1 + x_2$
$\sqrt{x_1} + x_2$

These utility fns aren't realistic but are easy to work with mathematically.

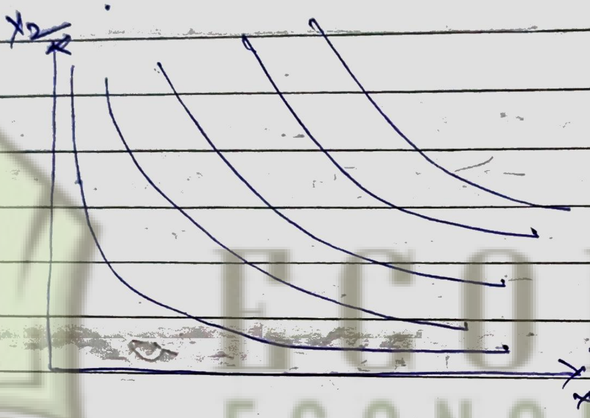
For convexity $v''(x_1) \leq 0$

(6) Cobb-Douglas preferences

The case (i) was a special case of C-D utility fn

$$u(x_1, x_2) = k x_1^{c} \cdot x_2^{d}$$

The IC's are nice convex monotonic ICs which are well behaved.



when $c = 1/2$, $d = 1/2$

The 2 ~~A very~~ prominent MT of this fn are the
(i) ln M.T
(ii) raising to $1/c+d$ M.T.

(i) $u(x_1, x_2) = x_1^{c} \cdot x_2^{d}$

$v(x_1, x_2) = \ln u(x_1, x_2) = c \ln x_1 + d \ln x_2$

(ii) $v(x_1, x_2) = u(x_1, x_2)^{1/c+d}$

$= x_1^{c/c+d} \cdot x_2^{d/c+d}$

let $a = \frac{c}{c+d}$ $\Rightarrow$ $v(x_1, x_2) = x_1^{a} \cdot x_2^{1-a}$