Date
Page

Chapter 2
Budget Constraint.

Economists assume that the consumer choose the best bundle of goods they can afford

Consumption bundle

indicated by (x_1, x_2) that tells us how much of good 1 and good 2 the cons. is choosing to consume. An imp ass. is that $x_1 \geq 0$, $x_2 \geq 0$.

The budget constraint, budget line and the budget set are three different, related concepts.

Budget constraint, Budget line and Budget set.

Budget constraint refers to the constraint: $P_1 x_1 + P_2 x_2 \leq m$ where P_1 denotes price of good 1 and P_2 denotes price of good 2 and m denotes the amt cons have to spend.

It is with respect to this constraint that the cons. maximizes the utility fn. $U = f(x_1, x_2)$

Remember that the Langrangean method can be applied here with F the obj fn being $U = f(x_1, x_2)$ and constraint G being $P_1 \cdot x_1 + P_2 \cdot x_2 \leq m$ with $x_1 \geq 0$ $x_2 \geq 0$

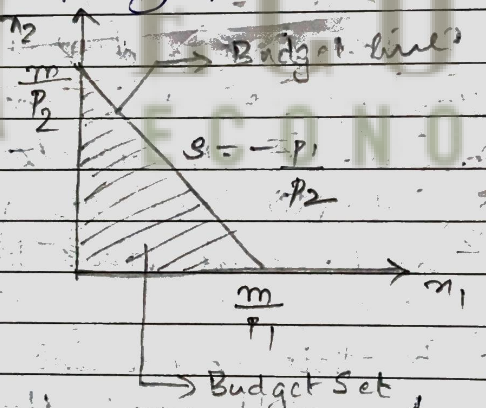
with $L(x_1, x_2) = F(x_1, x_2) + \lambda[g(x_1, x_2) - c]$

Here it is $U + \lambda[P_1 x_1 + P_2 \cdot x_2 - m]$.

Budget line shows all bundle of goods that the cons. is able to (or can) buy when they exhaust their entire income (here, m). It is shown by the eqn. $P_1x_1 + P_2x_2 = m$.

Budget Set consits of all bundles that are affordable at the given price & income. It is represented by the inequality $P_1x_1 + P_2x_2 \leq m$

It is to be noted that for all the defn., we look only 2 goods for convinience and so that a graph of the same can be made.



Slope of Budget line

Rearranging the eqn in the form $y = mx + c$ we get

$$\boxed{\frac{-P_1}{P_2}x_2 = \frac{-P_1}{P_2}x_1 + \frac{m}{P_2}}$$

Thus slope of the budget line is $-\frac{P_1}{P_2}$.
The slope tells us how much of one good the consumer has to give up for each addl unit of another good when the cons is spending all

income. It means the rate at which the mkt is willing to substitute good 1 for good 2.

Thus, $\boxed{\dfrac{dx_2}{dx_1} = \dfrac{-P_1}{P_2}}$

Suppose, the consumer is going to ↑ the cons of good 1 by Δx_1, then as the cons is using entire income, m, the consumption of good 2 has to be automatically reduced by (say) Δx_2

Thus eqn. $P_1 x_1 + P_2 x_2 = m$ ... (1)

becomes $P_1(x_1 + \Delta x_1) + P_2(x_2 + \Delta x_2) = m$ ... (2).

where $\Delta x_2 < 0$

(2) − (1)

$$P_1 \Delta x_1 + P_2 \Delta x_2 = 0$$

Thus $\dfrac{\Delta x_1}{\Delta x_2} = \dfrac{-P_1}{P_2}$

Also, the slope tells us the opp cost of consuming good 1 ie how much units of good 1 the cons has to give up for each addl unit of good 2.

Date
Page

Composite good interpretation

The 2 good assumptions can often be intepreted as one of the good representing everything else that the cons want to consume. Say, for example we need to know the cons behavior of a consumer's consumption of milk. Then x_1 represents no of units of milk ~~consumed~~ (litres of milk) consumed in a period of time (say a day) and x_2 representing all other comms other than milk that consumer consumes in a day. For obvious practical reasons the only common unit of measure of all other comm are with respect to the price i.e. in ₹, which means x_2 is the units of goods measured in ₹.

* Then P_2 'll be 1 as price of one INR is one

Thus budget constraint becomes

$$\boxed{P_1 \cdot x_1 + 1 \cdot x_2 \leq m}$$

This expression means the amt of money spend on good 1 $(P_1 \cdot x_1)$ & the amt spend on all other goods, x_2 must be no more than the amt of money the consumer has to spend $\Rightarrow m$.

* We say that the good 2 represents comp. good that stand for everything else that the consumer might want to

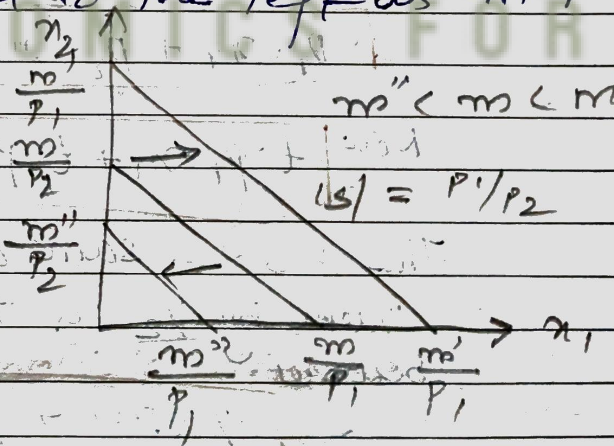
consume other good 1. Such a composite good is invariably measured in dollars (₹) to be spend on goods other than good 1]

Changes (change in slope & shifts) of the BL.

Changes in the budget line happens due to changes in income and prices.

Changes in Income :

If income m changes, cet paribus (i.e price ratio remain same). It doesn't affect the slope but it causes a parallel shift of the line to the right or to the left as $m \uparrow$ or $\downarrow$ respectively.

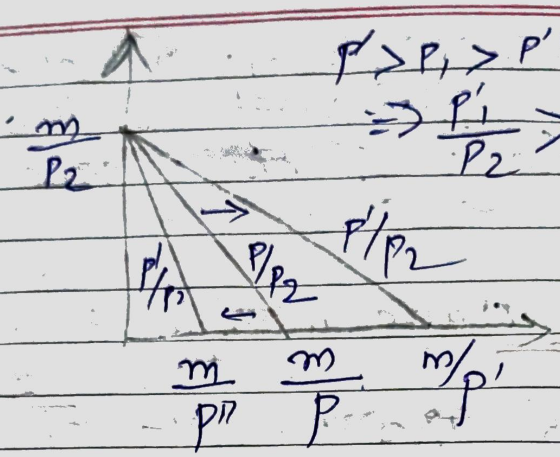


The steepness of the BL changes when the price of the commo changes, cet par (at a given income) as it affects the slope of the line.

If the price of good 1 $\uparrow$, cet par then BL will swivel inwards to the left with the same vertical intercept as before.
Similarly with the price of good 1 being reduced, the line becomes flatter as slope reduces.

$P' > P_1 > P''$

$\Rightarrow \frac{P'_1}{P_2} > P_1/P_2 > P''/P_2$



Simultaneous changes in the prices

if both prices of good 1 and 2 are say doubled, then that would mean ultimately, a shift of the budget line by 1/2

$$\boxed{p_1 \cdot x_1 + p_2 \cdot x_2 = m}$$

Now $\boxed{t \cdot p_1 x_1 + t \cdot p_2 x_2 = m}$ (here $t = 2$)

Thus it is the same as say reducing the income by 1/2 as the general eqn becomes

$$\boxed{p_1 \cdot x_1 + p_2 \cdot x_2 = \frac{m}{t}}$$

Thus it also means that if we reduce the income by t & the prices by t, then net effect is null.

What happens if the price change is not same?

if $P_2 \uparrow$ more than P_1, then $|s| = \frac{P_1}{P_2} \downarrow$
which makes BL flatter.

X Remember: A perfectly balanced inflation 'll've no effect on the cons. choices.

The Numeraire

$$\boxed{\begin{aligned} & P_1 \cdot x_1 + P_2 \cdot x_2 = m \\ \text{is same as } & \frac{P_1}{P_2} x_1 + 1 \cdot x_2 = \frac{m}{P_2} \quad (\div \text{ by } P_2) \\ \text{and } & \frac{P_1}{m} x_1 + \frac{P_2}{m} \cdot x_2 = 1 \quad (\div \text{ by } m) \end{aligned}}$$

We've pegged the price of good 2 to 1 in the first case & income to 1 in the second case.

But the eqn. remains unchanged.

The numeraire price is the price relative to which we are measuring the other price & income

classmate
Date
Page

Taxes, Subsidies & Rationing

Economic policies can affect the consumers budget constraint. A classic example is that of taxes & subsidies.

> Qty taxes.

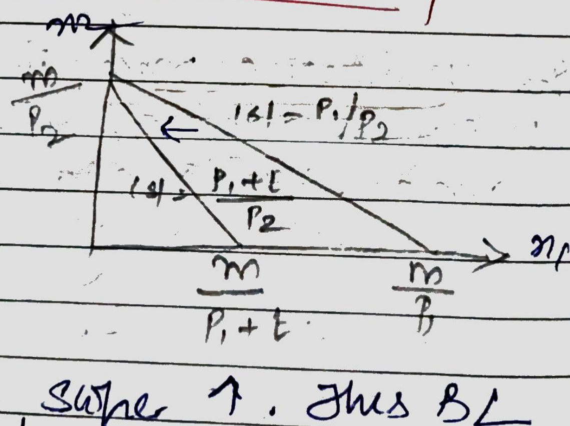
It is a situation where the cons has to pay a certain amt to the govt for each unit of the good that they purchase.

Say, for eg a tax of ₹ t per unit is imposed on the purchase of good 1. Then the price of good 1 'll simply change from P_1 to P_1+t for each unit. The cons 'ld be paying become P_1+t for each unit with P_1 paid to the seller & t paid to the govt.

Then the budget constraint

$$\boxed{(P_1+t)x_1 + P_2 \cdot x_2 = m}$$

$$\boxed{S = \frac{-(P_1+t)}{P_2}}$$



Absolute value of slope ↑. Thus BL would ~~shift~~ swivel inward along horizontal axis with the same vertical intercept $\left(\frac{m}{P_1}\right)$

It is because imposed tax is same as ↑ price of good.

$||^r$ argument can be made for a qty tax imposed on good 2 & for both goods simultaneously.

> Value taxes:-

It is also called as ad valorem taxes. It is a tax on the value i.e price of the good. It is usually expressed in % terms.

Say good 1 has a price of P_1 & is subjected to a sales tax @ τ.

Then the cons 'd be paying P_1 to seller & τ (%) of P_1 to the govt.

Thus, the price of each unit of good 1 'd ↑ to $(1+\tau) P_1$.

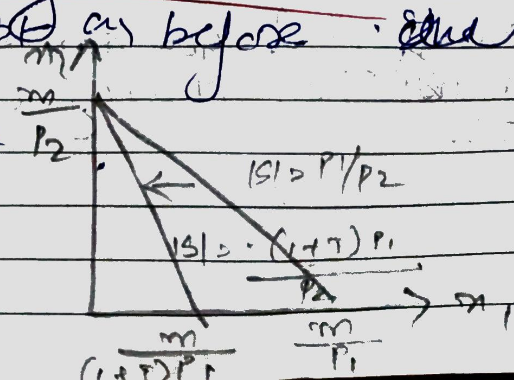
The budget line 'll be

$$[(1+\tau)P_1] \cdot x_1 + P_2 \cdot x_2 = m$$

$$\text{Slope} = \frac{-[(1+\tau) P_1]}{P_2}$$

The BL 'd be tilt inward with the same vertical intercept as before. ~~the image~~

Same image as before



Quantity subsidies –

The govt'ld given an amt to the cons on purchase of units of a subsidised comm say an amt of S.

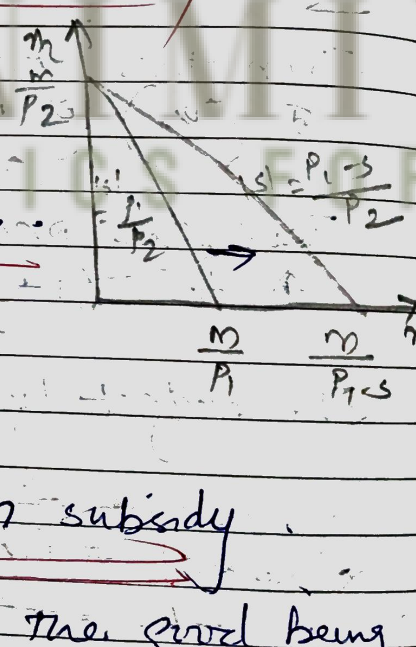
Then it is same as saying the cons can now pay just $(P_1 - s)$ for each unit of good 1 instead of P_1 if good 1 is the one that is subsidised.

BL becomes

$$(P_1 - S)x_1 + P_2 \cdot x_2 = m$$

$S \to P$ Slope $= \frac{-(P_1 - S)}{P_2}$

This'ld make the BL flatter.



Value subsidy / ad valorem subsidy.

It is based on the price of the good being subsidised (say good 1) If the govt gives you back every ₹1 back for every ₹2 you donate for charity. It means the charity is being subsidised at 50%.

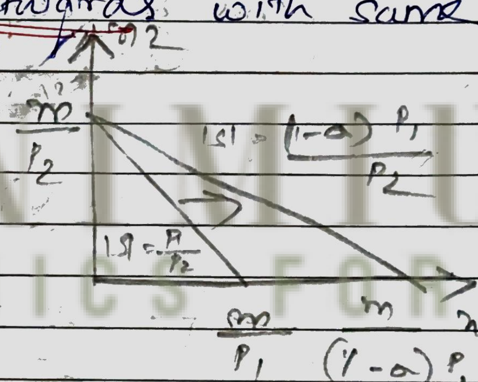
If the price of good 1 is subsidised at the rate of α (%), then the price of each unit of good becomes $(1-\alpha)P_1$.

Then BL is

$$\boxed{[(1-\alpha)P_1]x_1 + P_2 \cdot x_2 = m}$$

Slope = $\boxed{-\frac{(1-\alpha)P_1}{P_2}}$

The BL 'ld be tilting outwards with same vertical intercept as before.



> The case of lump sum Tax.

In this case, it means the govt takes away some fixed amt of money regardless of the individuals behaviour.

A lump sum tax 'ld mean a change in the real income (m) of the consumer & not the price.

Thus, the lump sum tax imposition of say T INR 'ld make the BL

$$\boxed{P_1 \cdot x_1 + P_2 \cdot x_2 = m - T}$$

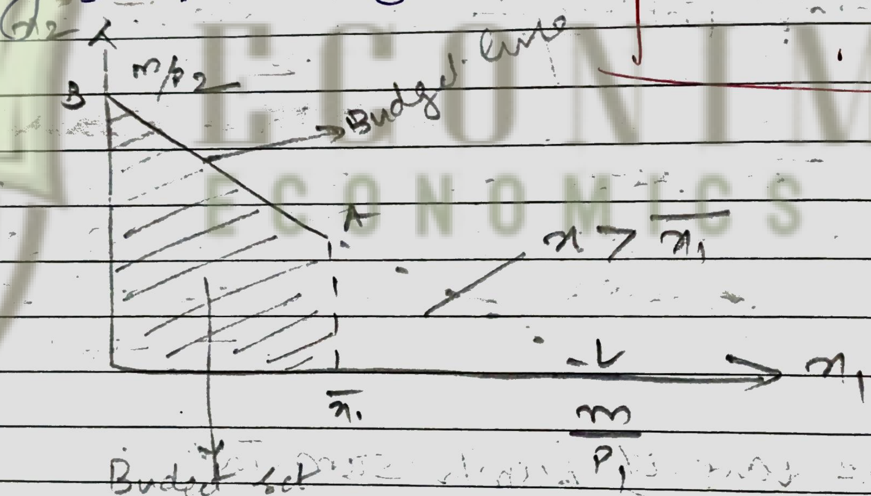
Thus BL shift inward

Rationing Constraints

This means that the level of cons of a good is fixed to be no more than some amt.

Suppose, good 1 were rationed so that no more than $\bar{x}_1$ could be consumed by a consumer.

Amt of commodity 2 left after $\bar{x}_1$ unit of cons of good 1 is given by $\boxed{x_2 = \dfrac{m - p_1\bar{x}_1}{P_2}}$



Remember that $A\bar{x}_1$ is not a part of the BL as it doesn't exhaust the consumer's income. As we go down along the line $A\bar{x}_1$, the amt of commo 2 reduces but the amt of comm 1 purchased remains the same due to rationing constraint. Thus, it is not a part of BL.

In case of rationing, the bundles beyond the rationed qty 'll be lopped of.

In this example the lopped off piece consists of all the cons bundles such that they're affordable within the rationg constraint. ~~This it is not a po~~ ie all those bundles where $x_1 > \bar{x}_1$

So the Lang prob 'll be

Maximise $F = u(x_1, x_2)$ s.t.

$$P_1 \cdot x_1 + P_2 \cdot x_2 \le m \text{ and } "x_1 > \bar{x}_1"$$

$$L(x_1, x_2) = F(x_1, x_2) + \lambda_1 [g_1(x_1, x_2) - c] + \lambda_2 [g(x_1, x_2) - c].$$

Here it is $u + \lambda_1 [P_1 x_1 + P_2 x_2 - m] + \lambda_2 [x_1 - \bar{x}_1]$

Combination of taxes, subsidies & rationing.

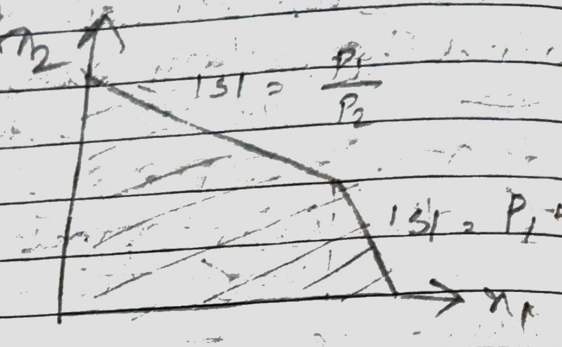
For example, a cons could consume good 1 at a price of P_1 up to $\bar{x}_1$ and then 'll 've to pay a tax t on all cons in excess of $\bar{x}_1$.

This is a combination of taxes & rationing.

Budget line 'll be such that it would have a kink at $\bar{x}_1$ with slope of the line to the left of the kink being $\frac{-P_1}{P_2}$ and to the right being $\frac{-(P_1 + t)}{P2}$.

Here t is a qty tax. If ad valorem tax was imposed, then the slope of the line to the ~~left~~ right of the kink be

$$-\left[\frac{(1+T)P_1}{P_2}\right]$$



Another example

A consumer could consume good 1 at a subsidised price of $P_1 - s$ up to x_0, and at a general price P_1 up to x_1 and then ~~it 4~~ for all consumption after x_1 'll be taxed at t.

Here the BL be s it 'll've kinks at x_0 & x_1.

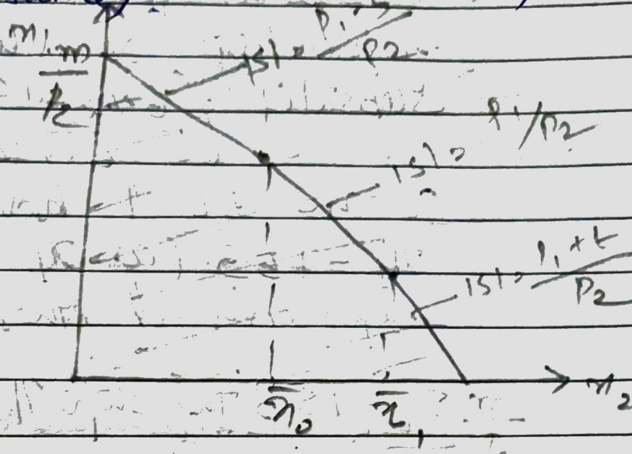
Slope ~~(S)~~ 'll be as followed

from 0 to x_0 $\rightarrow$ $\frac{-(P_1 - s)}{P_2}$

from x_0 to x_1 $\rightarrow$ = $\frac{-P_1}{P_2}$

Beyond x_1. $\rightarrow$ $\frac{-(P_1 + t)}{P_2}$

Remember that the taxes & subsidies mentioned are qty taxes. The slope'ld change if the goods are ad valorem, the image however would look similar (not exactly same).



Example of food stamp programme of US (pre 1979 case).

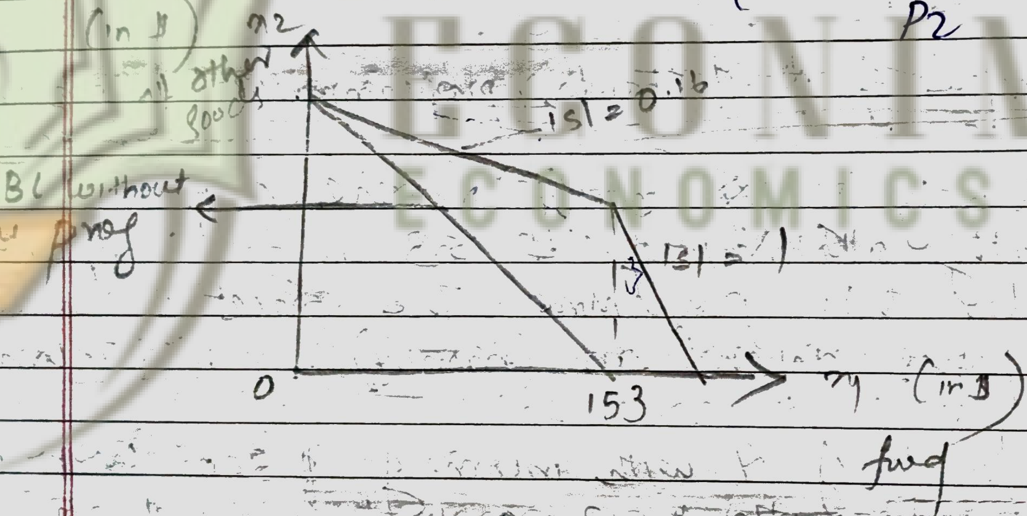
A family of 4 could receive a max monthly allotment of $ 153 in food coupons under this programme. The price of these coupons depended on (monthly) household income.

A family of 4 with income of $ 300 were allocated with $ 83 and that of $ 100 with $ 25 as payment.

That would mean say for second family for every $ 1 worth of food purchase, they'ld get $ 6.12 worth of food $\left(6.12 = \frac{153}{25}\right)$

In other words, it is a subsidy that means the family could purchase a food item worth $6.12 at a subsidised rate of $1.

Budget line eqn
$x_1 + x_2 = m$
As the household is allowed to buy \$153 of food stamps for \$25, that represents roughly an 84% of subsidy $\left(\frac{6.12-1}{6.12} \times 100 = 83.6 \approx 84\%\right)$
So the BL'll have a slope of -0.16 till $x_1 = 153$ (as x_1 represents food and x_2 represents all other items) and beyond $x_1 = 153$ the slope'll be -1 $\left(\text{as } \frac{-P_1}{P_2} = \frac{-1}{1} \Rightarrow 1\right)$.



» Let us take a general case -
Govt gives $\hat{x}$ amt of free food. Then the price of good 1 P_1 'll be 0 till $\hat{x}$. (BL eqn is $x_2 = m$). Thus making BL till that pt ∥el to horizontal axis. $\left(\frac{-P_1}{P_2} = \frac{0}{P_2} = 0\right)$ beyond the pt when the slope 'ld be $\frac{-P_1}{P_2}$