Equilibrium in terms of gross demand.

$$x_A^1(p_1^*, p_2^*) + x_B^1(p_1^*, p_2^*) = \omega_A^1 + \omega_B^1$$
$$x_A^2(p_1^*, p_2^*) + x_B^2(p_1^*, p_2^*) = \omega_A^2 + \omega_B^2.$$

Equilibrium in terms of net demand.

$$e_A^1(p_1^*, p_2^*) + e_B^1(p_1^*, p_2^*) = 0.$$
$$e_A^2(p_1^*, p_2^*) + e_B^2(p_1^*, p_2^*) = 0.$$

Agg excess dd $z_1 = e_A^1 + e_B^1.$
$z_2 = e_A^2 + e_B^2.$

Equilibrium in terms of agg net demand.

$$z_1(p_1^*, p_2^*) = 0$$
$$z_2(p_1^*, p_2^*) = 0.$$

Walras Law :- Value of agg excess dd is identically zero.

$$p_1 \cdot z_1(p_1, p_2) + p_2 z_2(p_1, p_2) \equiv 0.$$

Proof :-

$$p_1 \cdot x_A^1(p_1, p_2) + p_2 \cdot x_A^2(p_1, p_2) \equiv p_1 \omega_A^1 + p_2 \cdot \omega_A^2$$
$$p_1[x_A^1(p_1, p_2) - \omega_A^1] + p_2[x_A^2(p_1, p_2) - \omega_A^2] \equiv 0.$$
$$\Rightarrow \quad p_1 \cdot e_A^1 + p_2 \cdot e_A^2 \equiv 0. \quad — (1)$$
$$\text{III}^{ly} \quad p_1 \cdot e_B^1 + p_2 \cdot e_B^2 \equiv 0 \quad — (2)$$

(1) + (2)

$$\Rightarrow p_1(e_A^1 + e_B^1) + p_2(e_A^2 + e_B^2) \equiv 0.$$
$$\Rightarrow p_1 z_1 + p_2(z_2) = 0.$$

B If $z_1(p_1^*, p_2^*) = 0$, then $z_2(p_1^*, p_2^*)$ 'll always be automatically 0.

Suppose $z_1(p_1^*, p_2^*) = 0$ — (1)

Acc to Walras Law $p_1 z_1(p_1^*, p_2^*) + p_2 z_2(p_1^*, p_2^*) = 0$ } — (2)

(1) in (2)

$$\Rightarrow 0 \cdot p_2 \cdot z_2(p_1^*, p_2^*) = 0.$$

Assume $p_2 > 0$

then $z_2(p_1^*, p_2^*) = 0$

Demand functions:

<u>Cobb Douglas</u> $u = x_1^{c} \cdot x_2^{d}$.

$p_1 x_1 + p_2 x_2 = m$.

$x_1^* = \frac{cm}{(c+d)p_1}$, $x_2^* = \frac{d \cdot m}{(c+d)p_2}$

<u>Perf substitutes</u>

$u = x_1 + x_2$

$$x_1 = \begin{cases} m/p_1 & ; p_1 < p_2 \\ [0, m/p_1] & ; p_1 = p_2 \\ 0 & p_1 > p_2 \end{cases}$$

<u>Perf complements</u>

$u = \min(x_1, x_2)$

$x_1 = x_2 = \frac{m}{p_1 + p_2}$

<u>Quasilinear pref</u>. $u(x_1, x_2) = v(x_1) + x_2$

$= \ln x_1 + x_2$

$x_1 = \frac{p_2}{p_1}$.

$x_2 = \frac{m}{p_2} - 1$ if $\frac{m}{p_2} \geq 1$.

$= 0$ if $\frac{m}{p_2} < 1$.

Second Welfare Theorem.

If all agents have convex preferences, then there will always be a set of prices such that each pareto efficient allocation is a market equilibrium for an appropriate assignment of endowments.

$u_A : (x_A^1, x_A^2) = (x_A^1)^a \cdot (x_A^2)^{1-a}$.

$u_B \cdot (x_B^1, x_B^2) = (x_B^1)^b (x_B^2)^{1-b}$.

$$x_A^1 = a \frac{m_A}{P_1} \qquad x_A^2 = (1-a) \frac{m_A}{P_2}$$

$$x_B^1 = b \cdot \frac{m_B}{P_1} \qquad x_B^2 = (1-b) \frac{m_B}{P_2}$$

where $m_A = P_1 \omega_A^1 + P_2 \cdot \omega_A^2$.

$m_B = P_1 \cdot \omega_B^1 + P_2 \cdot \omega^2_B$.

Agg excess dd is :-

$$z_1(P_1, P_2) = a \cdot \frac{m_A}{P_1} + b \frac{m_B}{P_1} - \omega_A^1 - \omega_B^1.$$

$$= a \frac{P_1 \omega_A^1 + P_2 \omega_A^2}{P_1} + b \cdot \frac{P_1 \cdot \omega_B^1 + P_2 \cdot \omega_B^2}{P_1} - \omega_A^1 - \omega_B^1,$$

$$|||^{ly} \quad z_2 = (1-a) \frac{P_1 \omega_A^1 + P_2 \omega_A^2}{P_2} + (1-b) \frac{P_1 \omega_B^1 + P_2 \cdot \omega_B^2}{P_2} - \omega_A^2 - \omega_B^2.$$

Set $P_2 = 1$.

$$\Rightarrow z_1(P_1, 1) = a \cdot \frac{P_1 \omega_A^1 + \omega_A^2}{P_1} + b \cdot \frac{P_1 \omega_B^1 + \omega_B^2}{P_1} - \omega_A^1 - \omega_B^1 \quad -(1)$$

$$z_2(P_1, 1) = (1-a)(P_1 \omega_A^1 + \omega_A^2) + (1-b)(P_1 \omega_B^1 + \omega_B^2) - \omega_A^2 - \omega_B^2 \quad -(2).$$

put (2) = 0.

$$(1-a)(P_1 \omega_A^1 + \omega_A^2) + (1-b)(P_1 \omega_B^1 + \omega_B^2) - \omega_A^2 - \omega_B^2 = 0$$

$$P_1 \omega_A^1 + \omega_A^2 - a P_1 \omega_A^1 - a \omega_A^2 + P_1 \omega_B^1 + \omega_B^2 - b P_1 \omega_B^1 - b \omega_B^2 - \omega_A^2 - \omega_B^2 = 0$$

$$P_1 (\omega_A^1 - a \omega_A^1 + \omega_B^1 - b \omega_B^1) = \cancel{-\omega_A^2} + a \omega_A^2 \cancel{- \omega_B^2} + b \omega_B^2 \cancel{+ \omega_A^2} \cancel{+ \omega_B^2}.$$

$$P_1 \left[(1-a)\omega_A^1 + (1-b)\omega_B^1\right] = a \omega_A^2 + b \omega_B^2.$$

$$\Rightarrow P_1^* = \frac{a \omega_A^2 + b \omega_B^2}{(1-a)\omega_A^1 + (1-b)\omega_B^1} \quad //$$

9

cond

First welfare theorem.

The first welfare theorem follows that all market equilibria are pareto efficient i.e a competitive market will exhaust all of the gains from trade.

Proof by contradiction.

Suppose (x^1_A, x^2_A) and (x^1_B, x^2_B) were equilibrium bundles but are not pareto efficient. i.e there exist another set of bundles (y^1_A, y^2_A) and (y^1_B, y^2_B) such that

$$\left.\begin{aligned} y^1_A + y^1_B &= \omega^1_A + \omega^1_B \\ y^2_A + y^2_B &= \omega^2_A + \omega^2_B \end{aligned}\right\} \text{feasibility}$$

and

$$\left.\begin{aligned} (y^1_A, y^2_A) &> (x^1_A, x^2_A) \\ (y^1_B, y^2_B) &> (x^1_B, x^2_B) \end{aligned}\right\} \text{Pareto optimality}$$

We know (x^1_A, x^2_A) and (x^1_B, x^2_B) was choosen at a price which was affordable. Then if (y^1_A, y^2_A) and (y^1_B, y^2_B) are better, it must cost more than what that ~~was~~ can be afforded by A and B.

So,

$$P_1 y^1_A + P_2 y^2_A > P_1 \omega^1_A + P_2 \omega^2_A \quad \text{--- (1)}$$

$$P_1 y^1_B + P_2 y^2_B > P_1 \omega^1_B + P_2 \omega^2_B \quad \text{--- (2)}$$

(1) + (2)

$$\Rightarrow P_1 (y^1_A + y^1_B) + P_2 (y^2_A + y^2_B) > P_1 (\omega^1_A + \omega^1_B) + P_2 (\omega^2_A + \omega^2_B)$$

From feasibility eqn. we can see

$$P_1 (\omega^1_A, \omega^1_B) + P_2 (\omega^2_A + \omega^2_B) > P_1 (\omega^1_A + \omega^1_B) + P_2 (\omega^2_A + \omega^2_B)$$

which is a contradiction

Hence the assm of (x^1_A, x^2_A) and (x^1_B, x^2_B) being ~~not~~ pareto optimal isnt true.

Hence equilibrium necessarily implies pareto efficiency.

ect → conditions for Pareto ~~op~~ efficiency and competitive eq.

$$\max_{x^1_A, x^2_A, x^1_B, x^2_B} u_A(x^1_A, x^2_A)$$

such that

$$u_B(x^1_B, x^2_B) = \bar{u} \quad \text{- utility constraint.}$$

$$\left.\begin{aligned} x^1_A + x^1_B &= w^1_A + w^1_B \\ x^2_A + x^2_B &= w^2_A + w^2_B \end{aligned}\right\} \text{Resource constraints.}$$

$$L = u_A(x^1_A - x^2_A) - \lambda\left(u_B(x^1_B, x^2_B) - \bar{u}\right) - \mu_1\left(x^1_A + x^1_B - w^1_A - w^1_B\right) - \mu_2\left(x^2_A + x^2_B - w^2_A - w^2_B\right)$$

where λ - Langrage multiplier for utility constraint.
μ_i = Lagrange multiplier for resource constraint.

FOC :

$$\frac{\partial L}{\partial x^1_A} = \frac{\partial u_A}{\partial x^1_A} - \mu_1 = 0 \quad \text{—— (1)}$$

$$\frac{\partial L}{\partial x^2_A} = \frac{\partial u_A}{\partial x^2_A} - \mu_2 = 0 \quad \text{—— (2)}$$

$$\frac{dL}{\partial x^1_B} = -\lambda\frac{\partial u_B}{\partial x^1_B} - \mu_1 = 0 \quad \text{—— (3)}$$

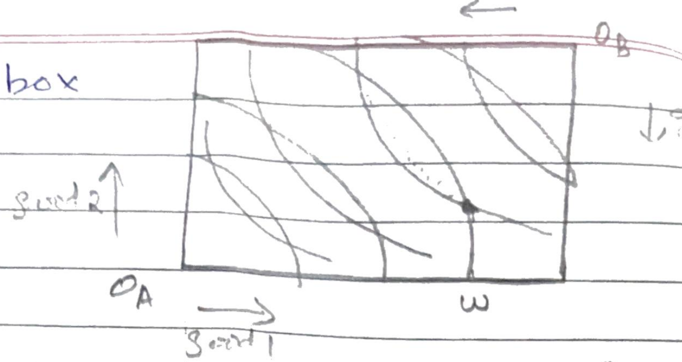
$$\frac{dL}{\partial x^2_B} = -\lambda\frac{\partial u_B}{\partial x^2_B} - \mu_2 = 0 \quad \text{—— (4)}$$

(1) ÷ (2) and (3) ÷ (4)

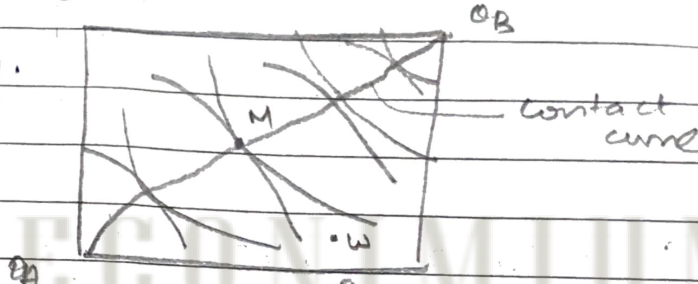
$$\left.\begin{aligned} \frac{\partial L/\partial x^1_A}{\partial L/\partial x^2_A} &= MRS_A = \frac{\partial u_A/\partial x^1_A}{\partial u_A/\partial x^2_A} = \frac{\mu_1}{\mu_2} \\ \frac{\partial L/\partial x^1_B}{\partial L/\partial x^2_B} &= MRS_B = \frac{\partial u_B/\partial x^1_B}{\partial u_B/\partial x^2_B} = \frac{\mu_1}{\mu_2} \end{aligned}\right\} \text{Pareto efficiency conditions.}$$

$$\left.\frac{\partial u_A/\partial x^1_A}{\partial u_A/\partial x^2_A} = \frac{P_1}{P_2}, \quad \frac{\partial u_B/\partial x^1_B}{\partial u_B/\partial x^2_B} = \frac{P_1}{P_2}\right\} \text{Competitive equilibrium conditions.}$$

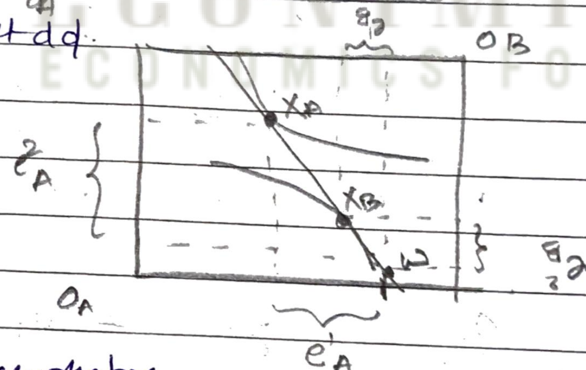
Edgeworth box



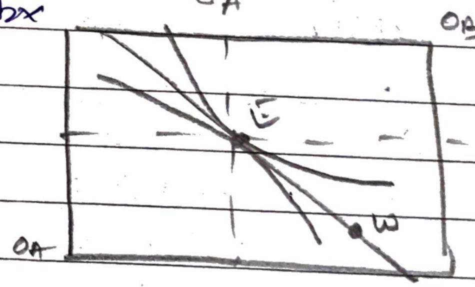
Pareto optimality.



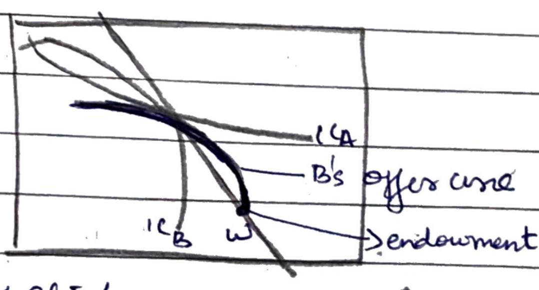
Gross dd Vs Net dd.



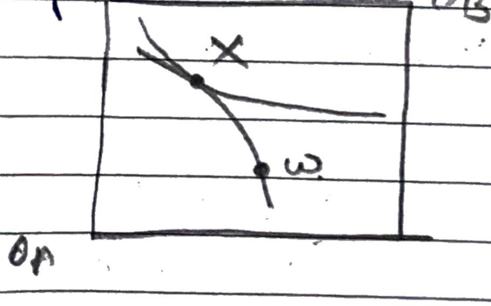
Equilibrium in edgeworth box



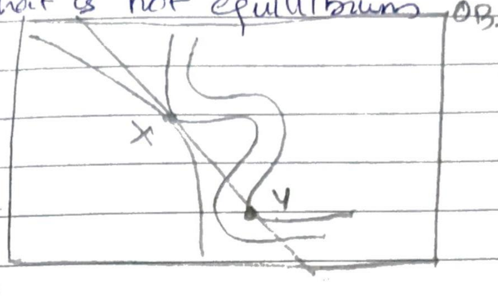
Monopoly in Edgeworth box



A perfectly descriminating monopolist.



Pareto efficient allocation that is not equilibrium



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Classical utilitarian or Benthamite welfare function :- $W(u_1, u_2, \ldots u_n) = \sum_{i=1}^{n} u_i$.

Weighted sum of utilities welfare function :- $W(u_1, \ldots u_n) = \sum_{i=1}^{n} a_i u_i$ $a_i > 0$ importance

Minimax or Rawlsian welfare function :- $W(u_1, \ldots u_n) = \min(u_1, \ldots u_n)$

One restriction - Increasing on individual consumer's utility.

Arrow's Impossibility Theorem.

Given these favorable properties :-

(1.) Given any set of complete, reflexive and transitive individual preferences, the social decision mechanism should result in preference that satisfy the same properties.

(2.) If everybody prefers alternative x to alternative y, then the social preferences should rank x ahead of y.

(3.) The preferences between x and y should depend only on how people rank x vs y and not on how they rank other alternatives.

If a social decision mechanism satisfies all these 3 properties, then it must be a dictatorship. all social rankings are the rankings of one individual.

Individualistic welfare function or Bergson - Samuelson welfare function :

$$w = w(u_1(x_1) \cdots , u_n(x_n))$$

Competitive equilibrium from equal division must be a fair allocation.

Proof

Suppose there are 2 agents A and B and two goods 1 and 2. Suppose they had an equal division allocation and then traded to a Pareto efficient allocation. i.e. they move to a new allocation where each agent is choosing the best bundle of goods he or she can afford at equilibrium prices (P_1, P_2). It must be already known that such an allocation is pareto efficient.

Suppose it is not equitable

Let A envies B

$$\text{ie } (x_A^1, x_A^2) <_A (x_B^1, x_B^2).$$

but (x_A^1, x_A^2) was the best bundle she could afford $\Rightarrow$ that (x_B^1, x_B^2) must cost more.

$$\Rightarrow P_1 \omega_A^1 + P_2 \omega_A^2 < P_1 x_B^1 + P_2 x_B^2$$

But this is a contradiction.

A and B started with exact same bundle since they started from an equal division!

Hence proved.

Hose max

elfare maximisation

B-S welfare fn

$$\max_{x_A^1, x_A^2, x_B^1, x_B^2} \; W\left(U_A(x_A^1, x_A^2), U_B(x_B^1, x_B^2)\right)$$

Such that $T(X^1, X^2) = 0$.

X^1 – total amount of 1 and X^2 – total amount of good 2 produced and consumed

$$L = W\left(U_A(x_A^1, x_A^2), U_B(x_B^1, x_B^2)\right) - \lambda\left(T(X^1, X^2) - 0\right)$$

FOC $\dfrac{\partial L}{\partial x_A^1} = \dfrac{\partial W}{\partial U_A} \cdot \dfrac{\partial U_A}{\partial x_A^1} - \lambda \dfrac{\partial T}{\partial X^1} = 0$ — (1)

$$\frac{\partial L}{\partial x_A^2} = \frac{\partial W}{\partial U_A} \cdot \frac{\partial U_A}{\partial x_A^2} - \lambda \frac{\partial T}{\partial X^2} = 0 \quad — (2)$$

$$\frac{\partial L}{\partial x_B^1} = \frac{\partial W}{\partial U_B} \cdot \frac{\partial U_B}{\partial x_B^1} - \lambda \frac{\partial T}{\partial X^1} = 0 \quad — (3)$$

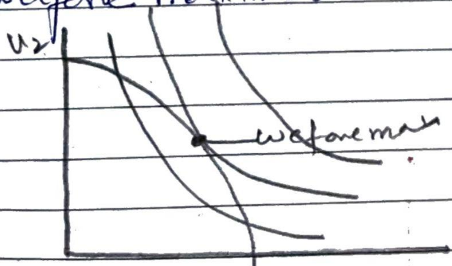
$$\frac{\partial L}{\partial x_B^2} = \frac{\partial W}{\partial U_B} \cdot \frac{\partial U_B}{\partial x_B^2} - \lambda \frac{\partial T}{\partial X^2} = 0 \quad — (4)$$

(1) ÷ (2) and (3) ÷ (4)

$$\Rightarrow \frac{\partial U_A / \partial x_A^1}{\partial U_A / \partial x_A^2} = \frac{\partial T / \partial X^1}{\partial T / \partial X^2}$$

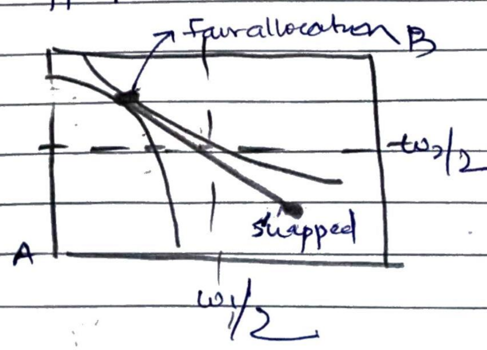
and $$\frac{\partial U_B / \partial x_B^1}{\partial U_B / \partial x_B^2} = \frac{\partial T / \partial X^1}{\partial T / \partial X^2}$$

welfare maximisation



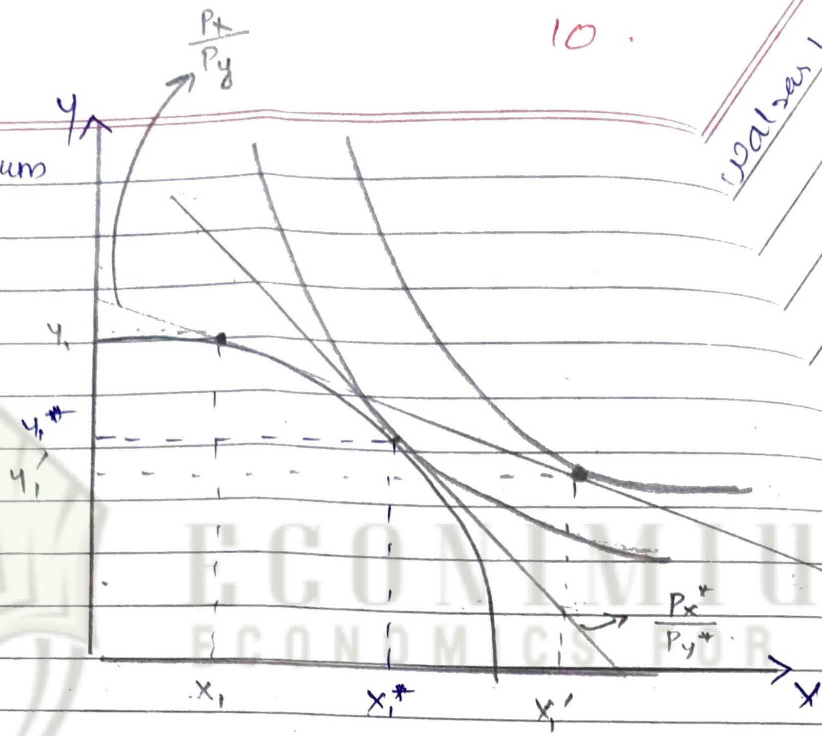
Straight isowelfare for weighted welfare fn

Swapped Fair allocation.



Walras La

(13) Equilibrium



Tech progress in X (take SE > IE)

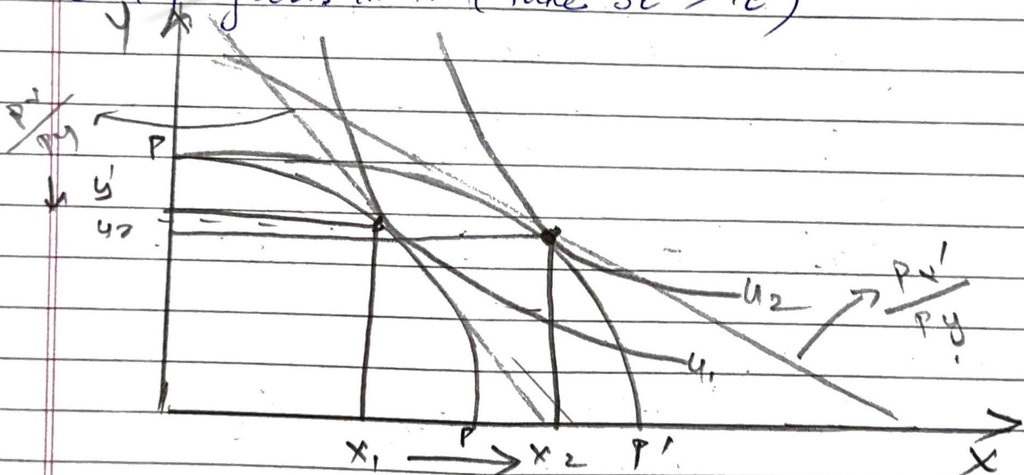
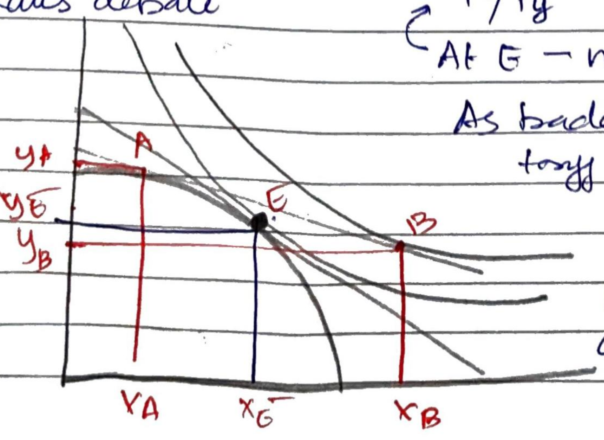


Figure with y ↑ (ie SE < IE) in text.

Corn Laws debate.



Px*/Py*

At E – no trade

As trade happens ToT & BL flatter @ Px'/Py'

Prod @ A

Cons @ B

Walras Law in vector form

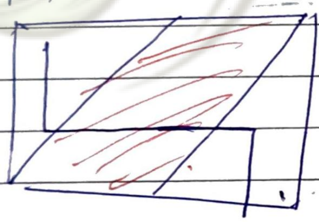
$$\sum_{i=1}^{m} x^i(p^*, p^*\bar{x}^i) = \sum_{i=1}^{m} \bar{x}^i$$

assume not all equilibrium prices can be zero.
dd fn are homogenous with degree 0

$x_1 + 2y_1$
$2x_2 + y_2$

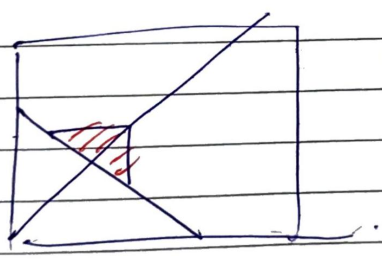
min.
min

$\cancel{x_1} = 0$
$y_2 = 0$



Set of all pts:
$x_1 \geq y_1$
$x_2 \geq y_2$

by sub
min



$x = y$

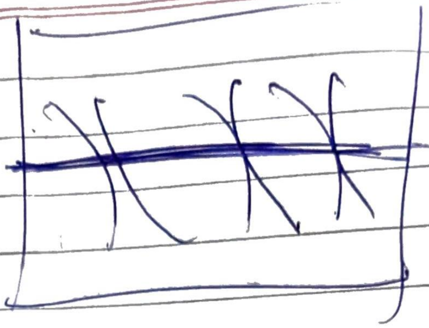
A $x_1 + 2y_1$
B $2x_2 + y_2$

Fair. A doesn't envy B $x_1 + 2y_1 > x_2 + 2y_2$

B doesn't envy A $2x_2 + y_2 > 2x_1 + y_1$

Quesilinear

for ~~linear~~ log



for ~~x, y~~ $u_1 = x_1 + 2\sqrt{y_1}$

$x_2 + 2\sqrt{y_2}$