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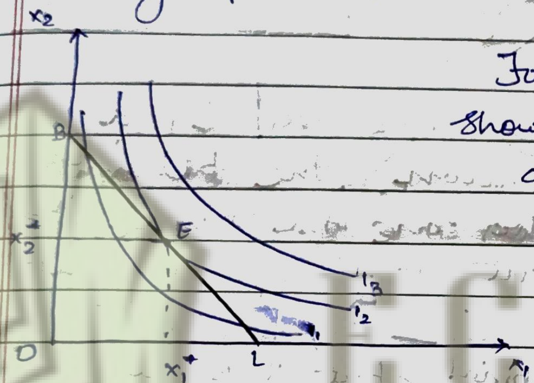
Unit 5
Choice

Optimal choice

The choice in which the consumer chooses the most preffered bundle from their budget set usually represented as $E(x_1^*, x_2^*)$.



For a typical case like the one shown here, E is the optimal choice where (x_1^*, x_2^*) maximises welfare.

Also the well behaved IC touches the budget line at a single point E.

Thus at equilibrium slope of IC at E = slope of the BL.

$$\boxed{MRS^E_{xy} = \frac{P_x}{P_y}}$$

* Remember that $x_1^* > 0$, $x_2^* > 0$.

* Note any choice beneath IC_2 that the consumer makes is affordable but is not welfare maximising. Any bundle beyond ~~the~~ I_2 is not affordable (until $m\uparrow$).

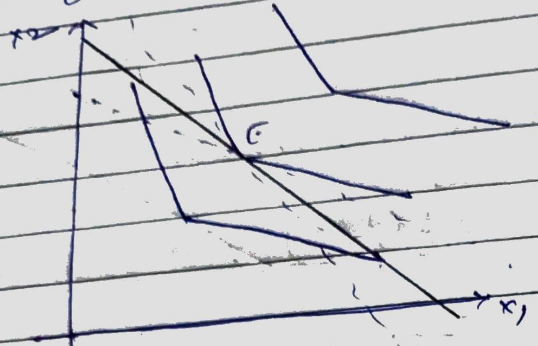
Is $MRS_{xy} = \frac{P_x}{P_y}$ a necessary condition?

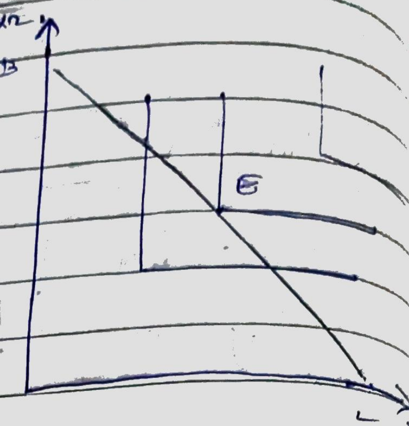
No. There are two exceptions.

Counter example 1 – The 'kinky' case

Counter example 2 – The case of boundary optimum.

Kinky case :-





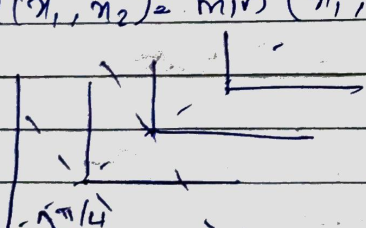
In cases where utility is represented by a curve with kinks as shown, the BL isn't "touching" the IC. In this case the mathematical defn of a unique tangent at E representing an optimal choice is disrupted.

Of course it is optimal, but it isn't a tangent that indicates $MRS_{xy} = P_x/P_y$.

A well known example is that of compliments where $U = \min(x_1, x_2)$.

Numerical expl.

$U(x_1, x_2) = \min(x_1, x_2)$. $\quad (P_1, P_2, M)$
$\quad (4, 8, 200)$.



$4x_1^* + 8x_2^* = 200$

$x_1^* = x_2^*$

$\Rightarrow 12x_1^* = 200$

$x_1^* = x_2^* = \frac{50}{3}$

Suppose assm $x_1^* > x_2^*$

$U(x_1^*, x_2^*) = \min(x_1^*, x_2^*)$

$= \min(50, 0)$

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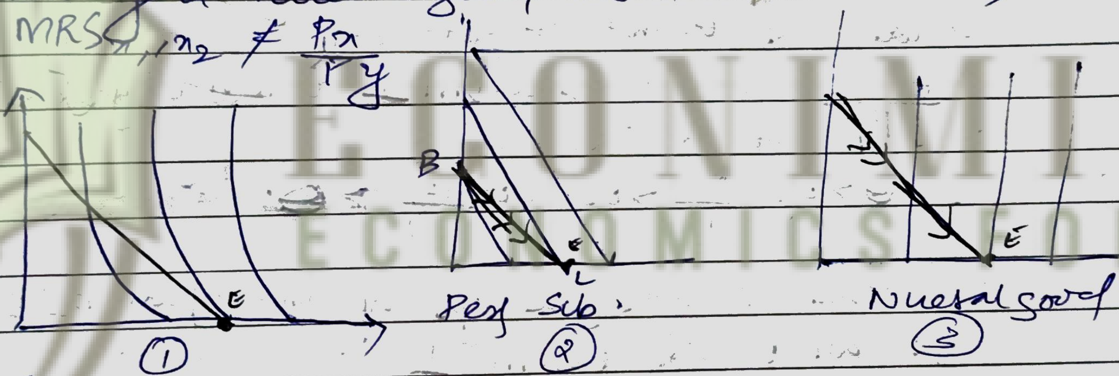
$\text{Min}(49, Y_2) = Y_2$.

$\frac{P_1}{P_2} = \frac{4}{8} = \frac{1}{2}$ //

The case of boundary optimum:

The typical case that we initially talked about is that of an interior optimum. There are cases where optimum choice is such that the consumer chooses a bundle such that no units of a good is consumed. A well known example is that of a neutral good and also perfect subst. Thus in that case,

$MRS_{x_1, x_2} \neq \frac{P_x}{P_y}$



Note: Here the budget lines doesn't cross ICs:

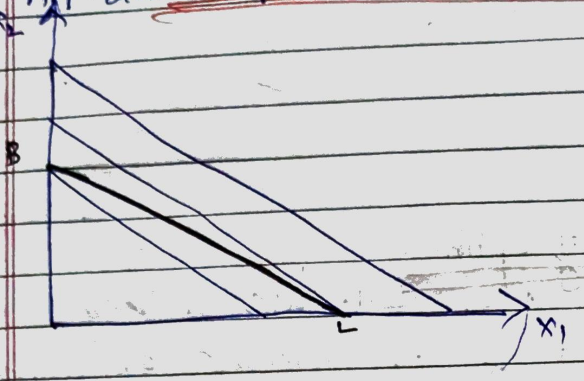
① It is a general case $MRS \neq P_x/P_y$ at E and it is clear that $x_2^* = 0$.

② $u(x_1, x_2) = 2x_1$ At L (E), $MRS \neq P_x/P_y$

③ $u(x_1, x_2) = x_1 + x_2$ (Perf Sub)

$MRS \neq P_1/P_2$.

An additional look into the perf substitute case



$u(x_1, x_2) = x_1 + x_2$

$MRS_{x_1 x_2} \neq \frac{P_1}{P_2}$

Here $P_2 > P_1$.

Utility fn says that give up $1 x_2$ to get $1 x_1$.

So if I have θ units of comm 2 then I can sell θ amt to get $\theta \cdot P_2$ to buy $\frac{\theta \cdot P_2}{P_1}$ amount of comm 1 which is greater than θ

$\theta > 0$ $\quad \frac{P_2}{P_1} > 0 \quad \theta \cdot \frac{P_2}{P_1} > \theta$

Thus the cr will give up all units of comm 2 and get all only of comm 1.

Old bundle

$u(x_1, \theta) = x_1 + \theta$.

New bundle

$$u\left(x_1 + \frac{\theta \cdot P_2}{P_1}, 0\right) = x_1 + \frac{\theta \cdot P_2}{P_1} + 0.$$

Clearly $x_1 + \theta \cdot \frac{P_2}{P_1} > x_1 + \theta$

$\left\{ \text{as } \frac{P_2}{P_1} > 1,\ \frac{P_2}{P_1} \cdot \theta > \theta,\ \frac{P_2}{P_1} \cdot \theta + x_1 > \theta + x_1 \right\}$

So if there is an utility like $u(x_1, x_2) = x_1 + x_2$ in which the consumer gives up goods on one-one basis, with $P_2 > P_1$. Therefore Cr will buy all of good 1 and 0 of good 2.

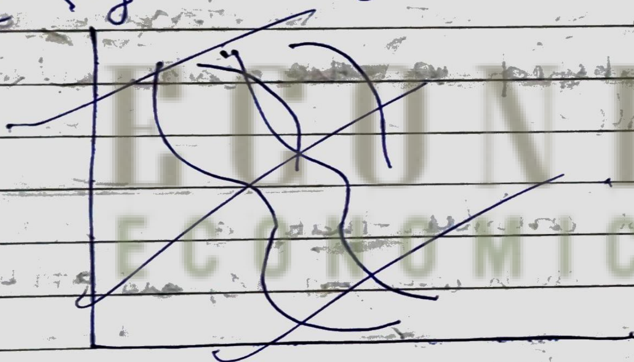
The conclusion that we can draw is if the optimum choice that we are talking about is ~~that~~ of an interior optimum, then necessarily the indifference curve 'll be tangent to the budget line ie $MRS_{xy} = P_x/P_y$

Remember when $P_1 = P_2$ IC coincides with BL.

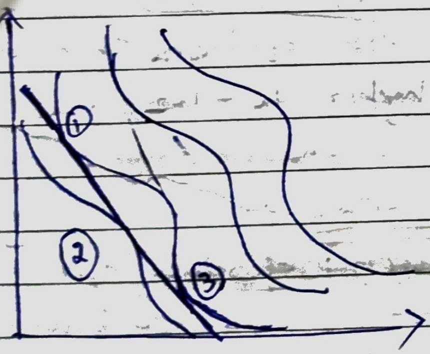
Is ~~θ~~ $MRS_{xy} = P_x/P_y$ sufficient condition ?.

Given by

Rulling out the case of boundary optimum & kinky cases, $MRS_{xy} = \frac{P_x}{P_y}$ is necessary, but not a sufficient condition



Multiple tangencies



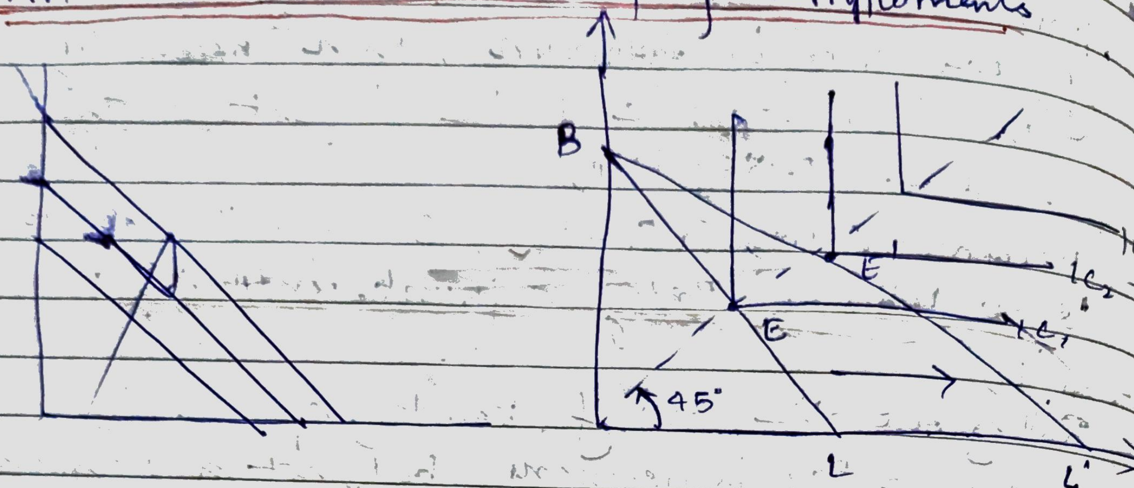
Here there are 3 tangencies, but only two optimal pts - thus the tangency condition is not sufficient.

Note

So "$MRS_{xy} = \frac{P_x}{P_y}$ condition that we drew above can be rephrased by adding the term "and sufficiently" iff we are talking abt convex preferences.

Convex pref with ~~θ~~ interior optimum ; then tangency is necessary and sufficient condition

An additional look into perf complements



We have seen that the tangency condition is not satisfied in the case of perf complements

Now just consider that @ (P_1, P_2, M) optimum was at E which was on IC_1 and ~~and~~ budget line BL.

With $P_1' < P_1$; Budget line swivells outwards to form BL'. Then the new equilibrium will be at E' (the new corner kink of a higher IC – IC_2).

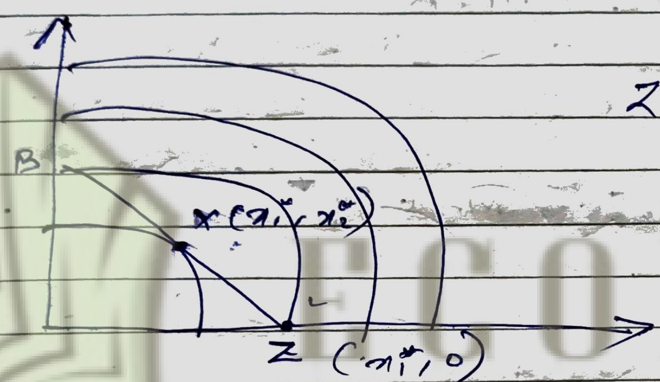
The case of concave preferences.

Concave preference ~~means~~ are such preference such that for any two bundles A and B, the average of A and B is worse than A. This is because concave ICs slope outwards and an average b/w 2 pts on the same IC would result in a point that is closer to the origin, thus giving a lower utility.

A combination of goods that can't be generally taken together are considered for such cases.

The optimal choice for these preferences are always a boundary point (where tangency condition isn't satisfied.

Q: If you've money to purchase ice cream and olives, and you dont like to consume them ~~together~~, you'll spend all your money on one / the other.



Z is the optimal choice, not X.

Choosing taxes:

We have already seen different types of taxes (in unit 2). Quantity tax and income tax are two of them. If the govt were to raise a certain amt of revenue via taxation which way is better?

Qty tax $-(t)$

Old BL $\quad p_1 x_1 + p_2 x_2 = m$

New BL $\quad (p_1 + t)x_1 + p_2 x_2 = m$.

Slope $= \dfrac{-(p_1 + t)}{p_2}$

The optimal choice (x_1^*, x_2^*) satisfies.

$$(p_1 + t)x_1^* + p_2 x_2^* = m$$

Thus revenue raised = $t \cdot x_1^*$ — (1)

Income tax ($m - R^*$ where $R^* = t x_1^*$)

old BL $p_1 x_1 + p_2 x_2 = m$

new BL $p_1 x_1 + p_2 x_2 = m - R^*$

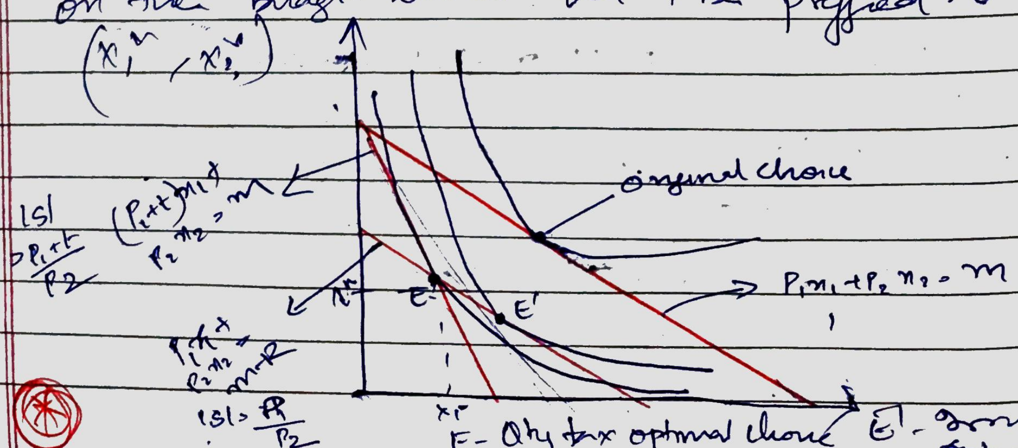
from (1) $R^* = t x_1^*$

$\Rightarrow p_1 x_1^* + p_2 x_2^* = m - t x_1^*$

slope $= -\dfrac{p_1}{p_2}$

The two new budget lines in each cases pass thru (x_1^*, x_2^*).
At (x_1^*, x_2^*) the MRS is $\dfrac{-(p_1+t)}{p_2}$. But

the income tax allows us to trade @ of exchange of $-\dfrac{p_1}{p_2}$. Thus the budget line cuts the indiff curve at (x_1^*, x_2^*), which implies that there'll be some pt on the budget line that'll be preffered to (x_1^*, x_2^*)



E - Qty tax optimal choice
E' - Income tax optimal choice

Turn pages to continue for limitations of this conclusion.

Deriving the demand functions

There are 2 broad ways, the classic ~~max~~ optimisation solution and the Langrangean method.

To explain these further, let us take the example of the Cobb Douglas preference.

We know $u(x_1, x_2) = x_1^c \cdot x_2^d$

we also know $p_1 x_1 + p_2 x_2 = m$

And we know at optimal choice MRS = Price ratio.

$$\boxed{MRS = -\frac{du/dx_1}{du/dx_2}} = -\frac{x_2^d \cdot c x_1^{c-1}}{x_1^c \cdot d x_2^{d-1}}$$

$$= -\frac{c}{d}\frac{x_2}{x_1}$$

Price ratio $= -p_1/p_2$

The first method can be proceded in two ways

Way 1 Solving for the tangency condition and BL constraint:

~~i.e. MRS = Price ratio~~

⇒

Solve $\frac{c}{d}\frac{x_2}{x_1} = \frac{p_1}{p_2}$ — (1)

$$p_1 x_1 + p_2 x_2 = m$$

$$x_2 = \frac{m - p_1 x_1}{p_2} \quad \text{— (2)}$$

(2) in (1)

$$\Rightarrow \frac{c\,((m - p_1 x_1)/p_2)}{d\,x_1} = \frac{p_1}{p_2}$$

$$\frac{c\cdot(m - p_1 x_1)}{\cancel{p_2}\cdot d x_1} = \frac{p_1}{\cancel{p_2}}$$

$$c(m - p_1 x_1) = p_1 \cdot d x_1$$

$$cm - cp_1 x_1 = d p_1 x_1$$

$$cm = d p_1 x_1 + c p_1 x_1$$

$$cm = x_1(d p_1 + c p_1)$$

$$\frac{cm}{(c+d)p_1} = x_1$$

||| ly $x_2 = \frac{dm}{(c+d)p_2}$

Way 2

Take monotonic form and then put our BL eqn

$u = x_1^c x_2^d$

$V = \ln u = c \ln x_1 + d \ln x_2$

$p_1 x_1 + p_2 x_2 = m$

$$x_2 = \frac{m - p_1 x_1}{p_2}$$

$$\Rightarrow V = c \ln x_1 + d \ln \left(\frac{m}{p_2} - \frac{p_1 x_1}{p_2} \right)$$

$$= \cancel{c \ln x_1 +}$$

$$\frac{dV}{dx_1} = 0$$

$$\Rightarrow c \cdot \frac{1}{x_1} + d \cdot \frac{\cancel{p_2}}{m - p_1 x_1} \cdot \left(0 - \frac{p_1}{\cancel{p_2}} \right)$$

$$\frac{c}{x_1} - \frac{d \cdot p_1}{m - p_1 x_1} = 0$$

$$c(m - p_1 x_1) - d \cdot p_1 \cdot x_1 = 0$$

$$cm - c p_1 x_1 - d \cdot p_1 \cdot x_1 = 0$$

$$cm = c p_1 x_1 + d p_1 x_1$$

$$cm = x_1 (c p_1 + d p_1)$$

$$x_1 = \frac{cm}{(c+d) p_1}$$

IIIly $x_2 = \dfrac{dm}{(c+d) p_2}$

The second method is to apply Lagrangean method.

we need to $\max u = x_1^c \cdot x_2^d$

s.t. $p_1 x_1 + p_2 \cdot x_2 = m$.

Note: Actually $x_1 \geq 0$, $x_2 \geq 0$ must also be a constraint but constraint for $G(x)$ in Lagrangean methods only take equality and hence the non-negative constraint is not introduced. This, however can make some conclusions abt p_1, p_2, x_1, x_2 (all that it may not be rational).

$u = x_1^c \cdot x_2^d$

mo. trans. $v = \ln u = c \ln x_1 + d \ln x_2$

$f(x) = c \ln x_1 + d \ln x_2$

$G(x) = p_1 x_1 + p_2 x_2 = m$.

$L(x, y) = f(x, y) + \lambda(-G(x, y) + m)$

$L(x_1, x_2) = c \ln x_1 + d \ln x_2 + \lambda(-p_1 x_1 - p_2 x_2 + m)$

$L'_{x_1} = \frac{c}{x_1} - \lambda p_1 = 0$

$L'_{x_2} = \frac{d}{x_2} - \lambda p_2 = 0$

$L'_{\lambda} = -p_1 x_1 - p_2 x_2 + m = 0$.

$$\lambda = \frac{c}{x_1 p_1} \Rightarrow \frac{x_1 p_1}{c} = \frac{x_2 p_2}{d}$$

$$\lambda = \frac{d}{x_2 p_2} \qquad x_1 = \frac{c}{d} \cdot \frac{p_2}{p_1} \cdot x_2$$

$$-\cancel{p_1} \cdot \frac{c \cdot p_2}{d \cdot \cancel{p_1}} \cdot x_2 - p_2 \cdot x_2 + m = 0$$

$$-\frac{c \cdot p_2 \cdot x_2}{d} - p_2 \cdot x_2 = -m$$

$$\frac{p_2\, c \cdot x_2}{d} + p_2 \cdot x_2 = m$$

$$x_2 \left(\frac{p_2 \cdot c}{d} + p_2 \right) = m$$

$$x_2 \times \frac{c \cdot p_2 + d \cdot p_2}{d} = m$$

$$x_2 \times \frac{p_2 (c+d)}{d} = m$$

$$\boxed{x_2 = \frac{dm}{p_2 (c+d)}} \;//$$

iii) $x_1 = \dfrac{cm}{(c+d)\, p_1}$ //

Limitations to the results of qty tax vs income taxes (pg 89)

(*) It only applies to one consumer. A uniform income tax is not necessarily better than a uniform quantity tax for all consumers as income tax'll typically differ from person to person.

(*) There is an underlying assn that when we impose the tax on income the consumer's income doesn't change. If income earned by the cr is taxed, one might expect that it'll discourage earning income so the after tax income might fall by even more than amt taken by the tax.

(*) In this case, we have left out the supply response to taxes.

Cobb Douglas preferences

The demand functions are:

$$x_1 = \frac{c}{c+d} \frac{m}{P_1}$$

$$x_2 = \frac{d}{c+d} \frac{m}{P_2}$$

The C.D pref have a prop :

Consider the fraction of consumer's income that is spend on good 1

$$\text{units consumed} = x_1$$

$$\text{cost} = p_1 \cdot x_1$$

$$\text{fraction of income} = \frac{p_1 x_1}{m} \quad \text{— (\#)}$$

put $x_1 = \frac{c}{c+d} \cdot \frac{m}{p_1}$ in (#)

$x_1^{1/2} \cdot x_2^{1/2}$ 250

$\frac{1}{2} \cdot 500$

5

30

$\frac{3}{4} \cdot 120$

$$\Rightarrow \frac{p_1 x_1}{m} = \frac{\cancel{p_1}}{\cancel{m}} \times \frac{c \cancel{m}}{(c+d)\cancel{p_1}}$$

$$\boxed{\frac{p_1 x_1}{m} = \frac{c}{c+d}}$$

$$\frac{90}{6} = 15$$

$$\text{||}^{ly} \quad \frac{p_2 x_2}{m} = \frac{d}{c+d}$$

Thus the Cobb Douglas consumer always spends a fixed fraction of his income on each good. The size of the fraction is determined by the exponent in the fn.

That is why its convenient to choose a representation of CD utility fn. in which exponents sum to 1

ie $u(x_1, x_2) = x_1^a x_2^{1-a}$. This we can immedtly interpret a as the fraction