Chapter 6
Demand

The consumer's demand functions gives the optimal amounts of each of the goods as a function of prices and income faced by the consumer

ie $$\boxed{\begin{aligned} x_1 &= x_1(p_1, p_2, m) \\ x_2 &= x_2(p_1, p_2, m) \end{aligned}}$$

Note: studying how a choice responds to changes in eco environment is known as comparative statics.

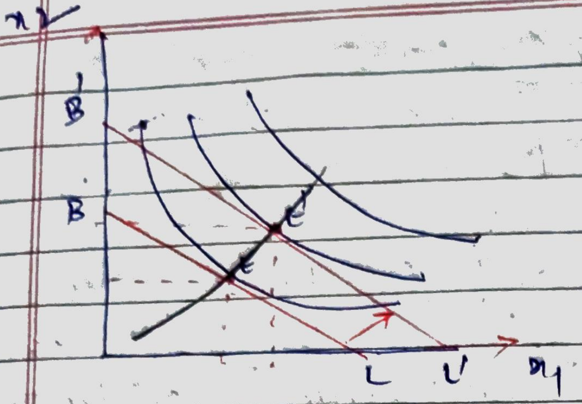
comparative in the sense, it compares two situations: before and after the change in the economic environment.

"Statics" means that we are not concerned with any adjustment process, it is just the result that matters (ie of choice here). This is what we do in this chap.

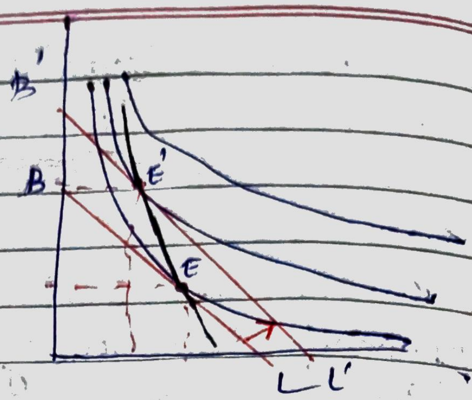
Normal and Inferior goods:

A good that has a +ve relation with income, cet. par. is normal good with $\boxed{\dfrac{dx(p_1, p_2, m)}{dm} > 0}$.

A good with -ve relation with income cet. par. is inferior good with $\boxed{\dfrac{dx(p_1, p_2, m)}{dm} < 0}$.



Normal goods



Inferior goods

Income offer curves

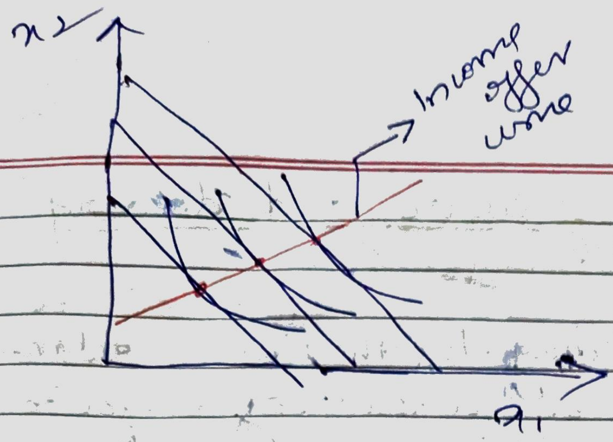
A line that connects all the equilibrium bundles that we get as we shift the budget line outwards is called income offer curve. It is also called as income expansion path.

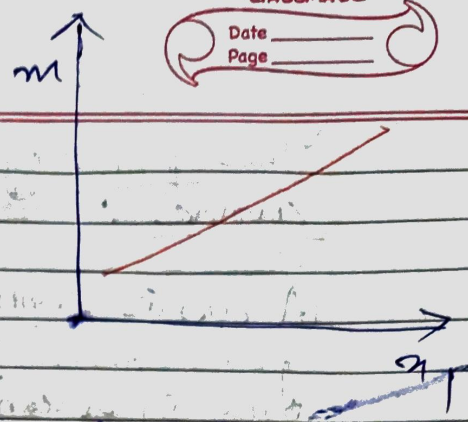
Income exp path is +ve a +ve slope for normal goods & -ve slope for inferior goods (represented by black lines in the top figures).

Engel curve.

Engel curve is a curve that shows the relationship between the qty demanded of a commodity with respect to changes in income only keeping all other things constant.

The engel curve is the locus of all equilibrium pt that is formed as a result of change in BL due to change in income.





Engel Curve.

~~Some examples.~~

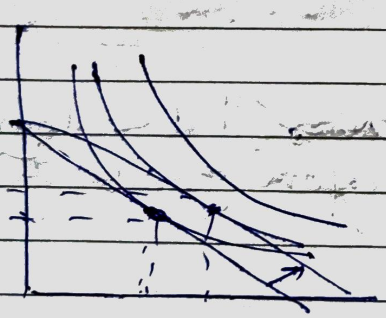
~~Cobb Douglas preferences.~~

Ordinary goods and Giffen goods.

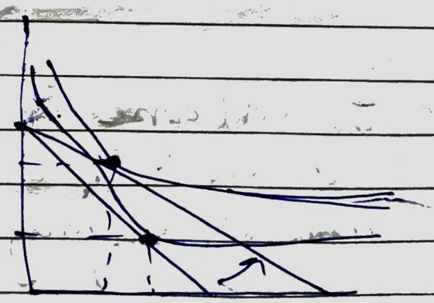
A good that has -ve relations with its own price is called ordinary good ie $\frac{d x_1(p_1, \bar{p}_2, \bar{m})}{dp_1} < 0$.

Otherwise ie when $\frac{dx_1}{dp_1} > 0$, it is called a giffen good.

That is when prices ↓, qty demanded to ↓.



Ordinary good



Giffen goods.

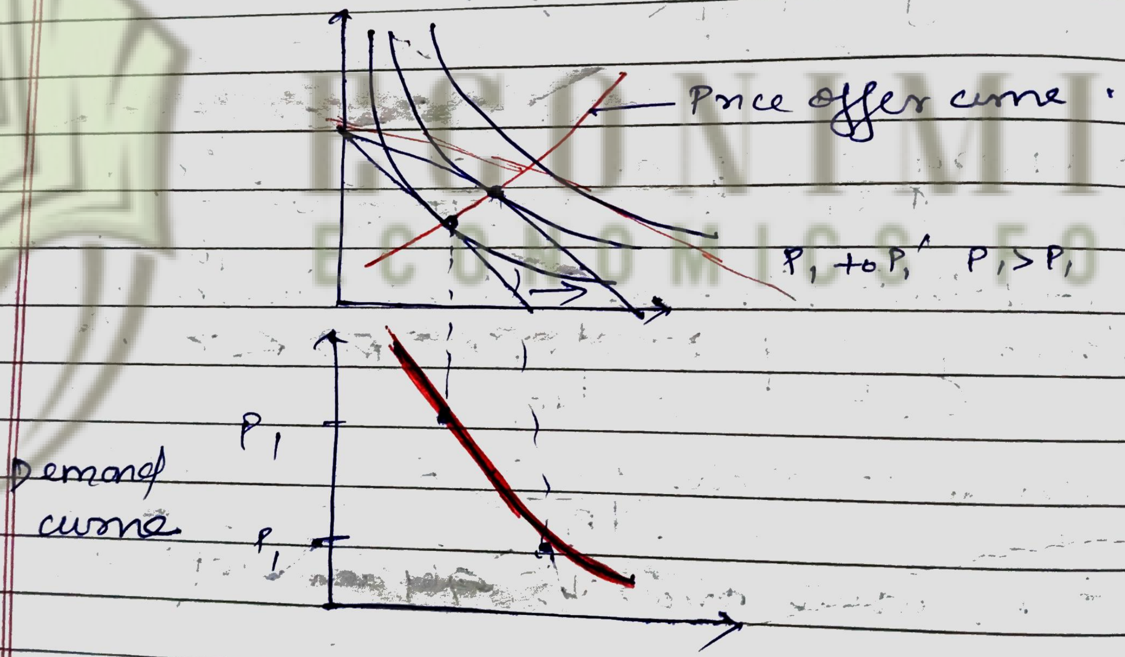
Refer Sem I note for more.

Refer Text for an eg.

The price offer curve and demand curve:

A curve connecting together all the optimal points formed as a result of changes in price of a commodity is called the price offer curve or price expansion path.

The curve drawn as a locus of all such equilibrium points i.e plotting the qty demanded of a commodity ~~keeping the~~ as prices changes.



Note: The dd curve in most cases'll be downward sloping. In case of giffen goods its upward sloping

Examples.

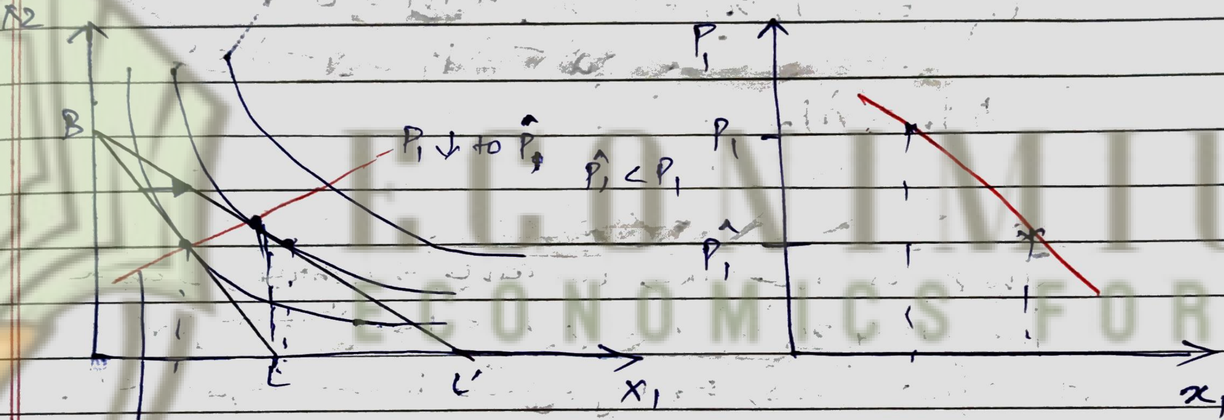
5) Cobb Douglas functions:

~~optimal points~~

Demand for $x_1^* = \dfrac{cm}{(c+d)p_1}$

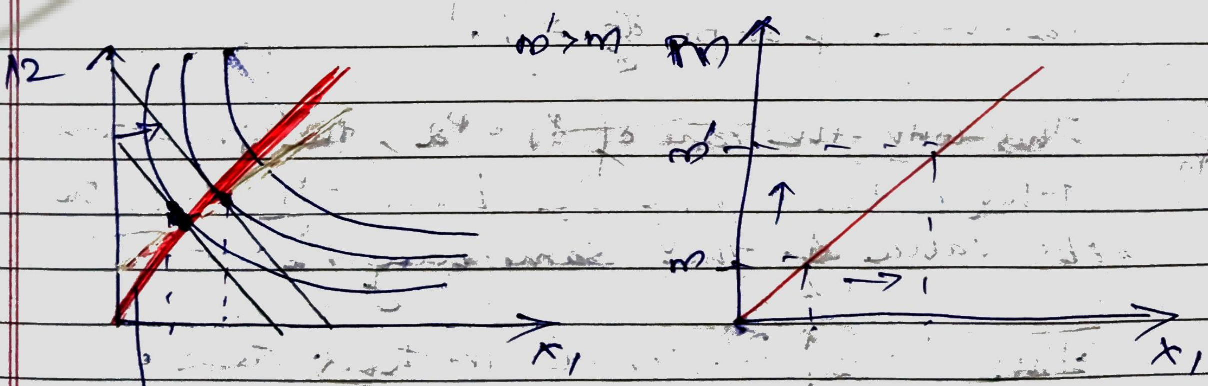
$x_2^* = \dfrac{dm}{p_2(c+d)}$

Refer previous chap for the calc.



→ Price exp path

Demand curve.



Income exp path

Engel Curve.

Perfect substitutes:

$$u(x_1, x_2) = \alpha x_1 + x_2$$

$$P_1 x_1 + P_2 x_2 = m$$

$$\mathcal{L} = \alpha x_1 + x_2 + \lambda(-P_1 x_1 - P_2 x_2 + m)$$

$$L'_{x_1} = \alpha - \lambda P_1 = 0$$
$$L'_{x_2} = 1 - \lambda P_2 = 0$$
$$L'_{\lambda} = -P_1 x_1 - P_2 x_2 + m = 0.$$

$$\alpha - \lambda P_1 = 1 - \lambda P_2$$

$$\lambda P_1 = \lambda P_2$$
$$P_1 = P_2$$

There is no way we can substitute x_2 as a function of x_1 in order to solve ~~the d~~ for dd fn. Thus, ~~wt~~ lagrangean method stops here.

We know that for perf substitute the abs. value of slope ~~of~~ $= 1$.

$\frac{P_1}{P_2} = 1$

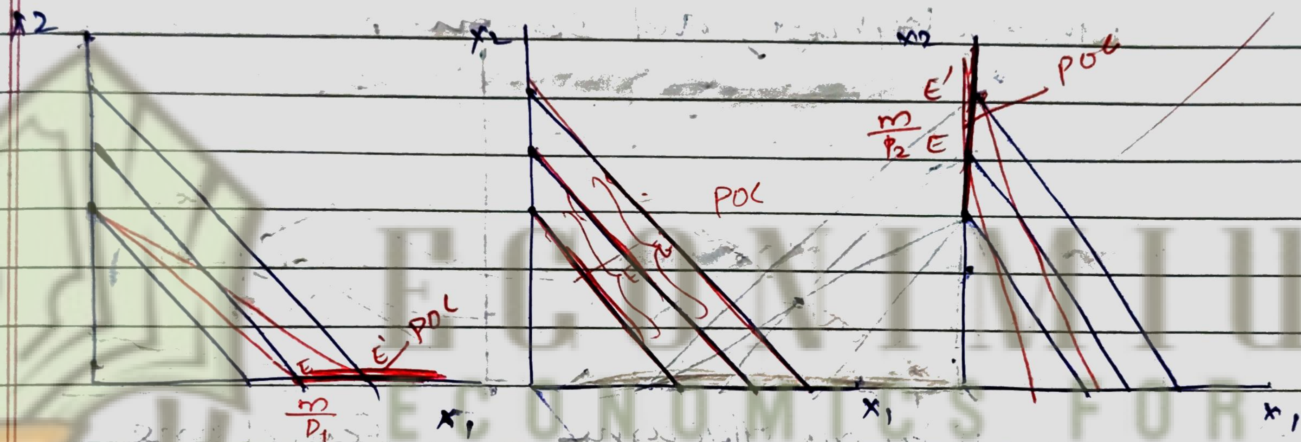
Thus with the case of $P_1 = P_2$, the price ratio which is the slope of BL becomes $-\frac{P_1}{P_2} = -1$, the abs value of the same being 1.

Thus IC and BL coincides in that case. Thus optimum choice is any bndls that lie on the BL/IC.

$\frac{P_1}{P_2} <$ If $\frac{P_1}{P_2} < 1$, ie $P_1 < P_2$ then the slope of the BL is flatter than that of ICs. Thus the consumer spends all income on good 1

$\frac{P_1}{P_2} > 1$ If $\frac{P_1}{P_2} > 1 \Rightarrow P_1 > P_2$, then the slope of the BL is steeper than that of ICs. Thus the consumer spends all income on good 2.

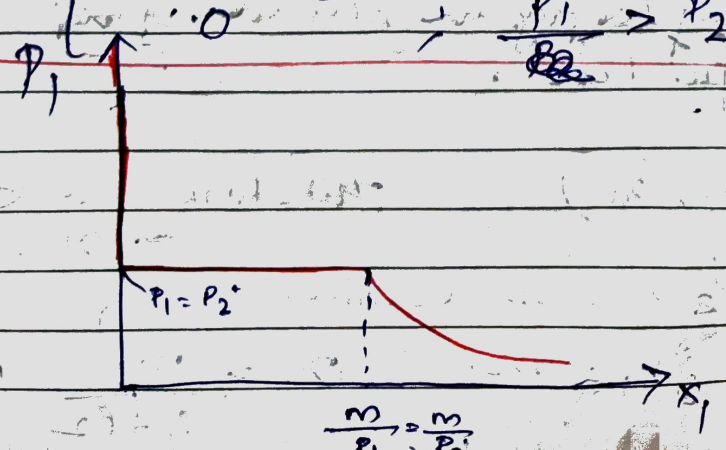
Price offer curve.



$\frac{P_1}{P_2} < 1$ $\frac{P_1}{P_2} = 1$ $\frac{P_1}{P_2} > 1$

Summing up the demand function would be:

$$x_1 = \begin{cases} m/P_1 & ; P_1 < P_2 \\ [0, m/P_1] & ; P_1 = P_2 \\ 0 & ; \frac{P_1}{P_2} > P_2 \end{cases}$$

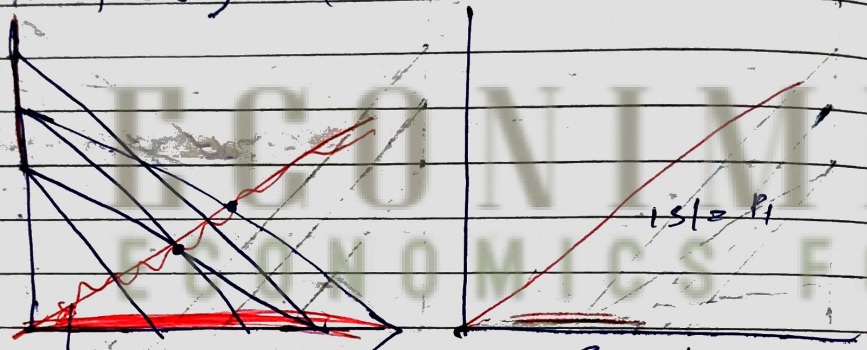


Demand Curve.

let $P_2 = 1$.

For all $P_1 > P_2$, 0 units of x_1 will be bought and that is shown by the vertical red line along Y axis (eg: Tea & coffee. Coffee is highly priced, ~~I opt to~~ I'll only opt tea in this price range)

As price comes down, there comes a point at which $P_1 = P_2$, where Ind curve is a horizontal line. (as units of coffee bought with tea kept const) and so on.



Income offer curve   Engel curve

7 Perfect complements:

$U = \text{Min}(x_1, x_2)$.

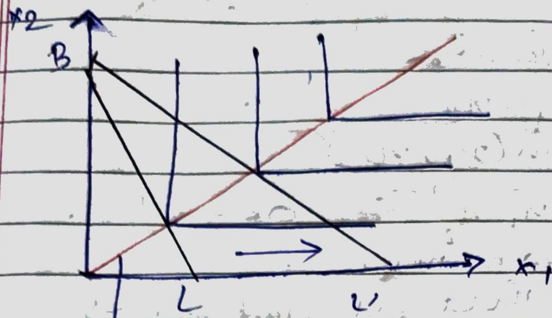
Without using Lagrangean, we can find the demand function,

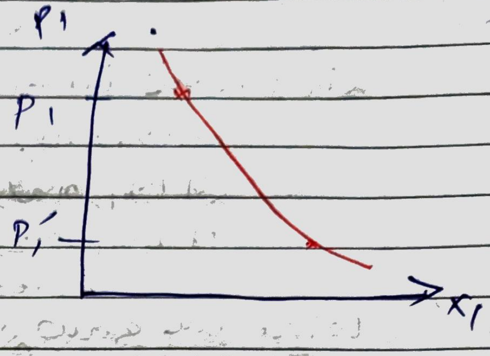
$U = \min(x_1, x_2)$ The kinks occur @ 45° inclination $\Rightarrow x_1 = x_2$

$\therefore P_1 x_1 + P_2 x_2 = m$ becomes

$P_1 x_1 + P_2 x_1 = m \Rightarrow x_1 = \dfrac{m}{P_1 + P_2}$ //

x_1, x_2 $\quad x_2 = \frac{m}{p_1+p_2}$



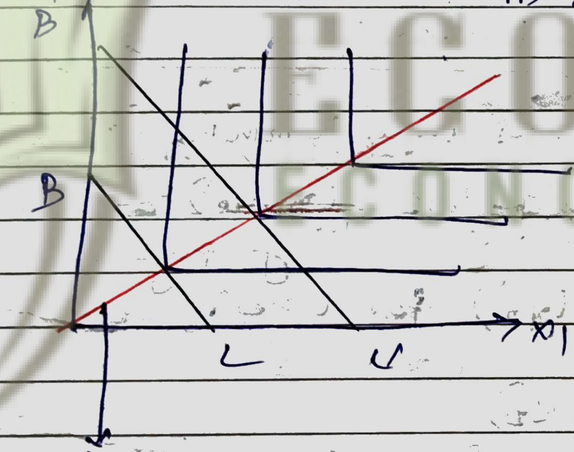


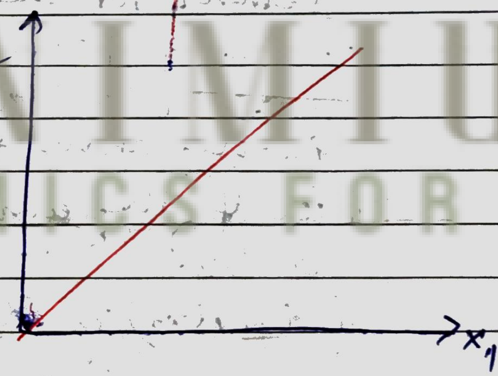
P_1 to P_1' $\quad P_1' < P_1$

Price exp path

Demand curve

m↑ to m' $\quad m' > m$





Income exp path

Engel curve

Homothetic preferences:

The previous three examples were

$u(x_1, x_2) = x_1 + x_2$

$u(x_1, x_2) = \text{Min}(x_1, x_2)$

$u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha} \quad 0<\alpha<1$.

common to them

What we saw was that all engel curves were straight upward sloping curves passing through the origin.

What does this curve imply? It implies that demand for the good went up at the same proportion as income.

All the above preferences were homothetic.

ie if the consumer prefers (x_1, x_2) to (y_1, y_2) then she automatically prefers (tx_1, tx_2) to (ty_1, ty_2) for $t>0$.

ie $(x_1, x_2) \succsim (y_1, y_2)$

$\Rightarrow (tx_1, tx_2) \succsim (ty_1, ty_2)$, $t>0$.

Take $u(x_1, x_2) = x_1 + x_2$

$u(y_1, y_2) = y_1 + y_2$

$u(tx_1, tx_2) = tx_1 + ty_1 \qquad t>0$.

$= t(x_1 + y_1)$

$u(ty_1, ty_2) = ty_1 + ty_2$

$= t(y_1 + y_2)$

we know $(x_1, x_2) \succsim (y_1, y_2)$ (let).

~~then~~ $tu(x_1, x_2) \succsim tu(y_1, y_2)$ (iff $t>0$),

$\Rightarrow$ $tx_1 + tx_2 \succsim ty_1 + ty_2$

$\therefore$ Homothetic.

$u(x_1, x_2) = \text{Min}(x_1, x_2)$ $\quad$ $(10, 20) \times 5$

$u(y_1, y_2) = \text{Min}(y_1, y_2)$ $\quad$ $(2, 4) \times 5$.

we know (say) $\text{Min}(x_1, x_2) \succsim \text{Min}(y_1, y_2)$,

then clearly $t \cdot [\text{Min}(x_1, x_2)] \succsim t\,[\text{Min}(y_1, y_2)]$

i.e. $\text{Min}(tx_1, tx_2) \succsim \text{Min}(ty_1, ty_2)$

$t > 0$,

$\therefore$ Homothetic.

$u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$.

$u(tx_1, tx_2) = t^{\alpha} x_1^{\alpha} \cdot t^{1-\alpha} \cdot x_2^{1-\alpha}$

$= t \cdot x_1^{\alpha} x_2^{1-\alpha} = t \cdot u(x_1, x_1)$

^{lly} $u(y_1, y_2) = y_1^{\alpha} \cdot y_2^{1-\alpha}$

$u(ty_1, ty_2) = t \cdot y_1^{\alpha} y_2^{1-\alpha} = t \cdot u(y_1, y_2)$

Let $u(x_1, x_2) \succsim u(y_1, y_2)$

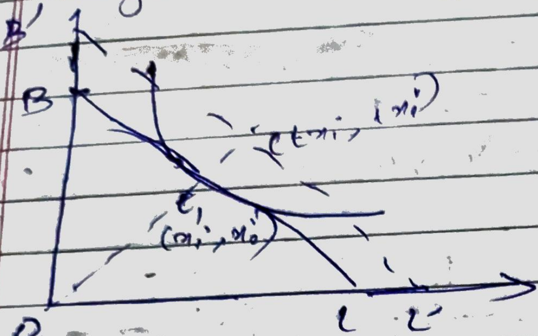
clearly $t \cdot x_1^{\alpha} \cdot x_1^{1-\alpha} \succsim t \cdot y_1^{\alpha} \cdot y_2^{1-\alpha}$

$\Rightarrow t\, u(x_1, x_2) \succsim t \cdot u(y_1, y_2)$

$u(tx_1, tx_2) \succsim u(ty_1, ty_2)$

$\therefore$ Homothetic

* Prove that if preferences are homothetic then engel curve is an ↑ sloping straight line passing through origin.



note
take $x_1 \neq x_2$
ie engel curve is not at $45°$ inclination

For (P_1, P_2, M) the budget set is OBL,
opt bundle = $E(x_1^*, x_2^*)$.

Let $M \uparrow$ to αM $\alpha > 0$

For $(P_1, P_2, \alpha M)$ if we are able to show that E_2 is at $\alpha x_1^*, \alpha x_2^*$ then proof is done

Claim 1: $\alpha x_1^*, \alpha x_2^*$ is affordable at $(P_1, P_2, \alpha M)$

Since (x_1^*, x_2^*) is the demanded bundle when the consumer faces (P_1, P_2, M)

$$P_1 x_1^* + P_2 x_2^* \leq M$$

Multiply by α, $\alpha > 0$ on both side,

$$\Rightarrow P_1(\alpha \cdot x_1^*) + P_2 \cdot (\alpha \cdot x_2^*) \leq \alpha \cdot M$$

Thus affordable.

Claim 2 : We know that (x_1^*, x_2^*) is the demanded bundle at (P_1, P_2, M).

$$u(x_1^*, x_2^*) \geq u(y_1, y_2) \quad \forall (y_1, y_2) \in OBL \text{ other than } (x_1^*, x_2^*).$$

$\Rightarrow (x_1^*, x_2^*) \succsim (y_1, y_2)$ where $(y_1, y_2) \in OBL$.

Then $(\alpha x_1^*, \alpha x_2^*) \succsim (\alpha y_1, \alpha y_2)$ where $(\alpha y_1, \alpha y_2) \in OB'L'$.

($\because$ pref are homothetic).

Proof.

Note:-

> If the dd for a good goes up by a greater proportion than income, we say it is a luxury good. If it goes up by a lesser proportion than income, we say it is a necessary good

> Homotheticity essentially means that when income is scaled up or down by any amt $t > 0$, the demanded bundles scales up or down by the same amt.

If the IC is tangent to the budget line at (x_1^*, x_2^*) then IC thr (tx_1^*, tx_2^*) is tangent to budget line that has t times as much income and the same prices.

Quasilinear preferences:
we know quasilinear pref are such that these ICs are shifted versions of each other.

$$u(x_1, x_2) = v(x_1) + x_2$$

$$u(x_1, x_2) = \ln x_1 + x_2$$

$$p_1 x_1 + p_2 x_2 = M$$

$$\ln x_1 + x_2 + \lambda(-p_1 x_1 - p_2 x_2 + M) = 0$$

$$L'_{x_1} = \frac{1}{x_1} - \lambda p_1 = 0$$

$$L'_{x_2} = 1 - \lambda p_2 = 0$$

$$L'_{\lambda} = p_1 x_1 + p_2 x_2 - M = 0$$

$$\frac{1}{x_1 p_1} = \lambda$$

$$\frac{1}{p_2} = \lambda$$

$$\frac{1}{x_1 p_1} = \frac{1}{p_2}$$

$$x_1 = \frac{p_2}{p_1}$$

Now $\cancel{p_1} \cdot \frac{p_2}{\cancel{p_1}} + p_2 \cdot x_2 - M = 0$

$$x_2 = \frac{M - p_2}{p_2} = \frac{M}{p_2} - 1$$

Now $P_1 > 0$ $P_2 > 0$

Then $x_2 = \frac{M}{P_2} - 1$ can undergo some interpretation

if $\frac{m}{P_2} < 1$ then $x_2 < 0$, not possible

This contradiction arrived because we have not taken the non negative inequality constant into consideration for this Lag part. Since it is an inequality we can't include it

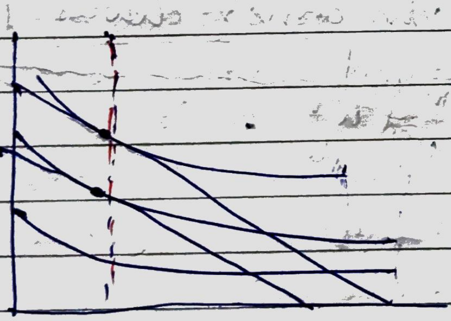
So an ~~anot~~ other assumption is to be laid out

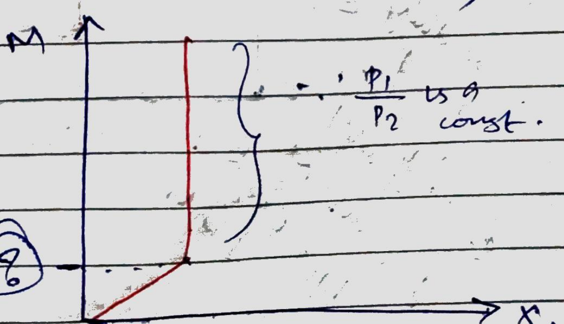
ie $\frac{M}{P_2} \geq 1 \Rightarrow x_2 = \frac{M}{P_2} - 1$

$\frac{m}{P_2} < 1 \Rightarrow x_2 = 0$

So ~~for~~ all pts where M (income) < P_2 (price of x_2)

$x_2 = 0$.





pg.
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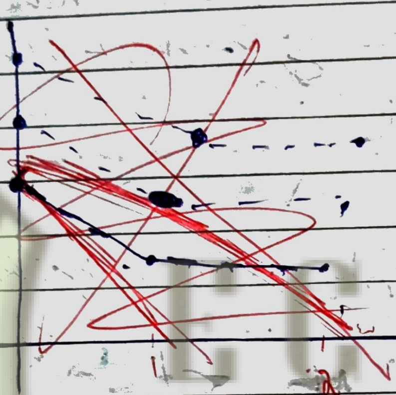
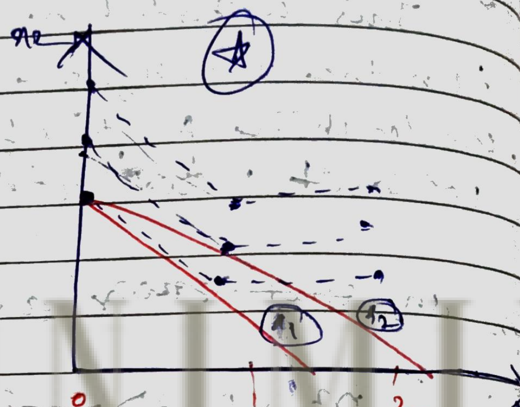
Engel curve

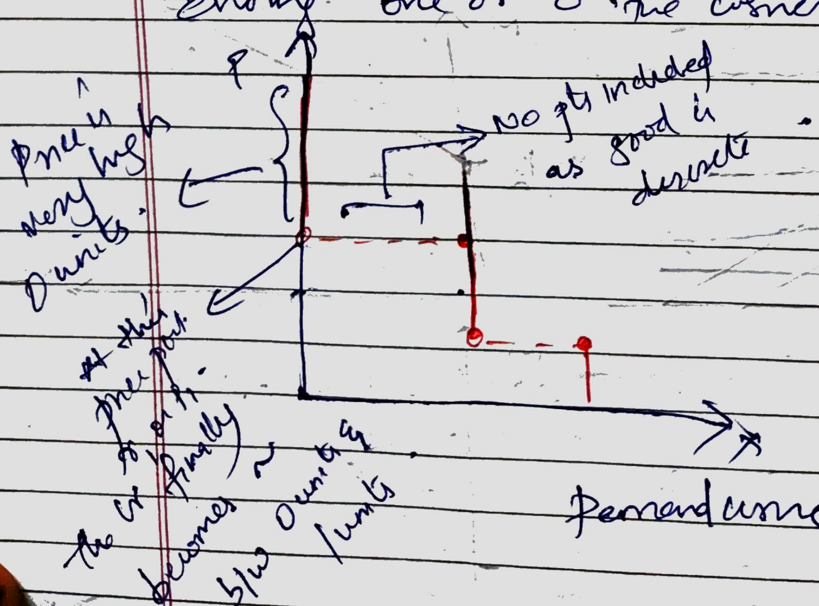
Discrete goods

Discrete goods are those goods which is consumed only in whole no. (ie. 1, 2 etc). (2.1 units isn't quite in alignment with the notion of disc. goods)



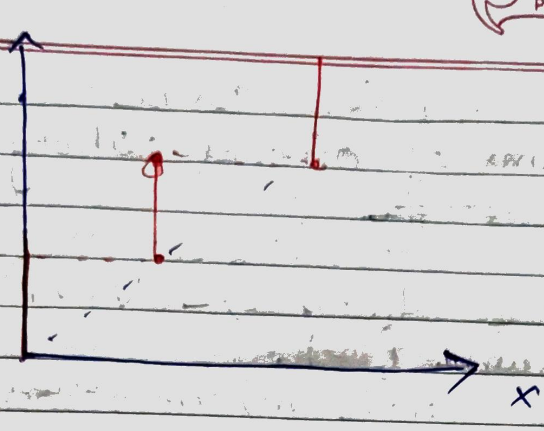


Goods can only be bought in whole no (dark dots). At initial price P_1 the corresponding budget set is such that optimum bundle is $(0, x_2)$. When price of good 1 reduced to such a price whereas the consumer is indifferent b/w choosing one or 0, the consumer chooses 1.

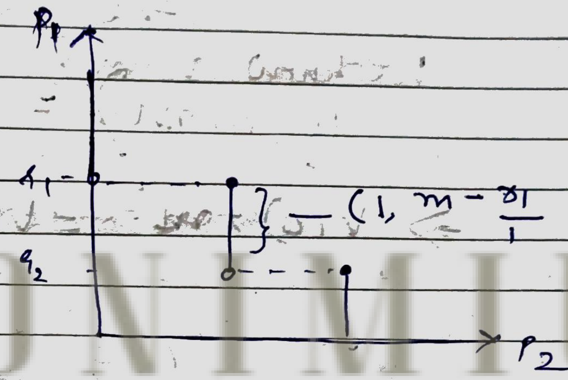


Demand curve.

Engel curve



Reservation price



r_1 is the highest price at which the consumer is willing to give up some units of good 2 for a unit of good 1.

r_t is the pt at which the cr finally becomes ~ b/w ~~per~~ ~~having~~ purchasing 0 units of good 1 or 1 unit of good 1 and it is assumed that the cr chooses latter.

Thus the set of such prices follow the sequential behavior:

$$\boxed{\text{Set} = \{ r_1, \ldots r_t \} \quad \& \quad r_t \leq m}$$

r_t is the max amt the cr is willing to give up to purchase the t^{th} unit.

As long as we have the assumption that preference are convex, the set of reservation price act in a manner s.t.

$$r_1 > r_2 > \ldots > r_t$$

(graphically slope of BC becomes flatter & flatter.

Consider the fig 4

These prices can be described in terms of original utility function.

~~For example~~.

Take $U(x_1, x_2) = v(x_1) + x_2$,

Assume $p_2 = 1$.

Take (p_1, p_2, m) combination

1.

We know

$$u(0, m) = u(1, m - r_1).$$

$$\Rightarrow v(0) + m = v(1) + m - r_1.$$

$$\Rightarrow r_1 = v(1) - v(0) = MU$$

When it comes to r_2

$$u(1, m - r_2) = u(2, m - 2r_2)$$

$$\Rightarrow v(1) + m - r_2 = v(2) + m - 2r_2$$

$$\Rightarrow r_2 = v(2) - v(1) = MU.$$

IIIly $r_3 = v(3) - v(2) = MU$.

Thus all of these price segments add utility the consumer is getting by consuming the next unit.

∵ we know $v'(x) > 0 \Rightarrow MU_1 > 0, MU_2 > 0, MU_3 > 0$

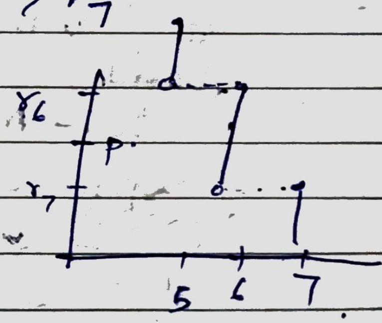
$\Rightarrow r_1 > 0, r_2 > 0, r_3 > 0$

$v''(x) < 0 \Rightarrow r_1 > r_2 > r_3$ (Convexity of preferences)

We noted that in each case, the reservation price measures the increment in utility necessary to induce the consumer to choose an add'l unit of the good.

Because of the special structure of quasilinear utility function, the reservation prices do not depend on the amt of good 2 that the cr has.

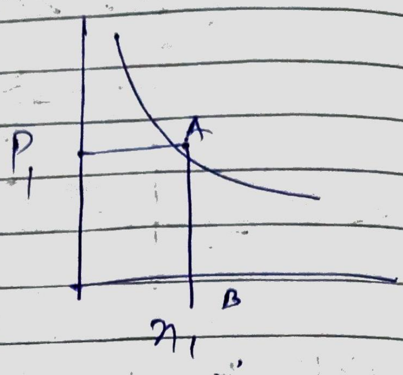
Suppose we take a price p s.t. $r_6 > p > r_7$



$r_6 > p$ ∴
The consumer is willing to give up p dollars per unit bought to get 6 units of good 1 and $p > r_7$ means the consumer is not willing to give p dollars per unit to get 7th units.

(A mathematical argument shown - refer pg 110-111)

Inverse demand fn



Just like how qty demanded as a function of price tells us about the direct demand curve,

there can be another interpretation where the height AB represents the max. value that a cr puts for buying x_1 units when we consider price as a function of demand, called as inverse dd fn.

Note:-

Consider x_1, x_2 unit of good1 and good2 where good2 means all other goods other than good1,

then MRS ~~becomes~~ means the money that cr spent on other goods other than good1 gives up for an addl unit of good 1.

Thus MRS becomes marginal willingness to pay

$$U = \alpha x_1 + \beta x_2$$
$$P_1 x_1 + P_2 x_2 - m = 0$$

$$\alpha x_1 + \beta x_2 + \lambda(-P_1 x_1 - P_2 x_2 + m) = 0$$

$$L'_{x_1} = \alpha - \lambda P_1 = 0$$
$$L'_{x_2} = \beta - \lambda P_2 = 0$$

$$L'_{\lambda} = -P_1 x_1 - P_2 x_2 + m = 0$$

$$\alpha = \lambda P_1 \qquad \alpha/P_1 = \lambda$$
$$\beta = \lambda P_2 \qquad \beta/P_2 = \lambda$$

$$\frac{\alpha}{P_1} = \frac{\beta}{P_2}$$

$$P_1 = \frac{\alpha}{\beta} P_2$$

$$\alpha > 0,\ \beta > 0 \Rightarrow \frac{\alpha}{\beta} > 0 \Rightarrow \frac{\alpha}{\beta} \cdot P_2 > 0$$